BINARY SEARCH TREES

Problem Solving with Computers-II

```
#include <iostream>
using namespace std;

int main(){
    cout<<"Hola Facebook\n";
    return 0;
}
```

My office hours are today!
(Mon) 2:30p to 3:30p in HFH 1155

Binary Search

- Binary search. Given `value` and sorted array `a[]`, find index `i` such that `a[i]` = `value`, or report that no such index exists.
- Invariant. Algorithm maintains `a[lo]` ≤ `value` ≤ `a[hi]`.

- Ex. Binary search for 33.

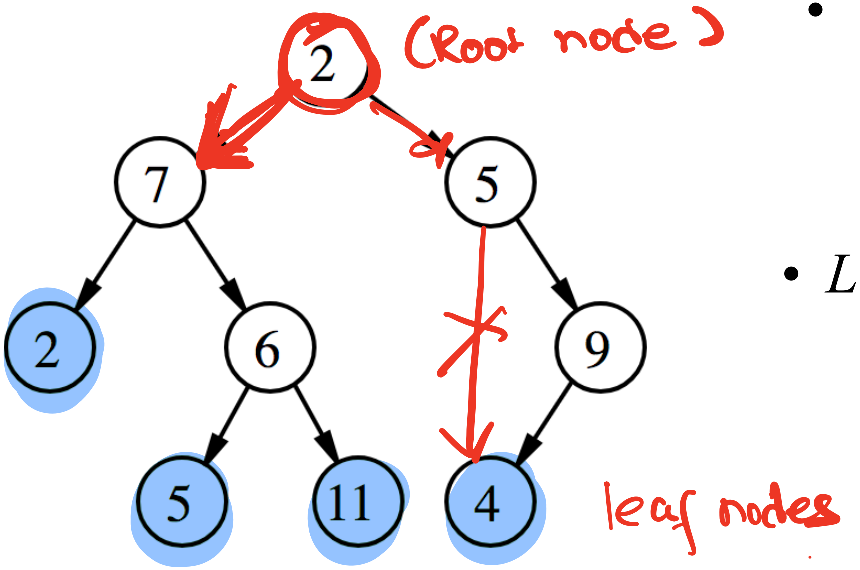
6	**13**	**14**	**25**	**33**	**43**	**51**	**53**	**64**	**72**	**84**	**93**	**95**	**96**	**97**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
↑														↑
lo														**hi**

Trees

A tree has following general properties:

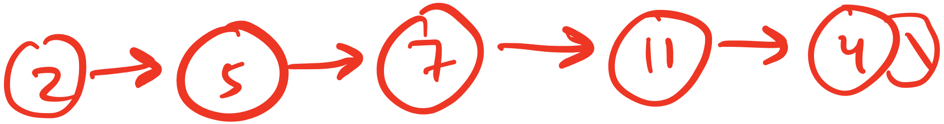
- One node is distinguished as a **root**;
- Every node (exclude a root) is connected by a directed edge *from* exactly one other node;

  A direction is: *parent -> children*
- *Leaf node: Node that has no children*



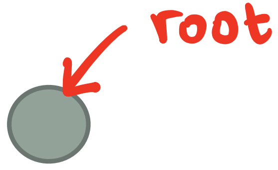
2 is the root node

2's children are 7 & 5

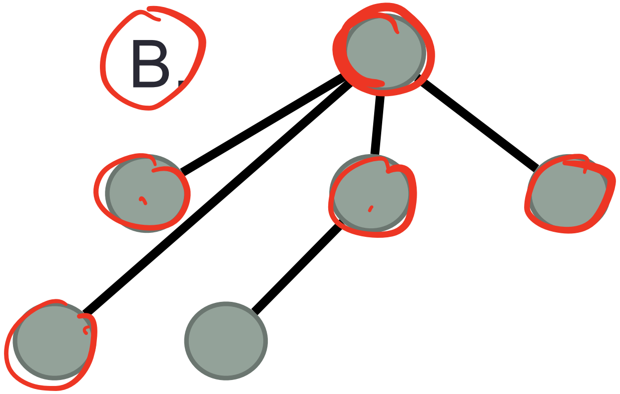


Which of the following is/are a tree?

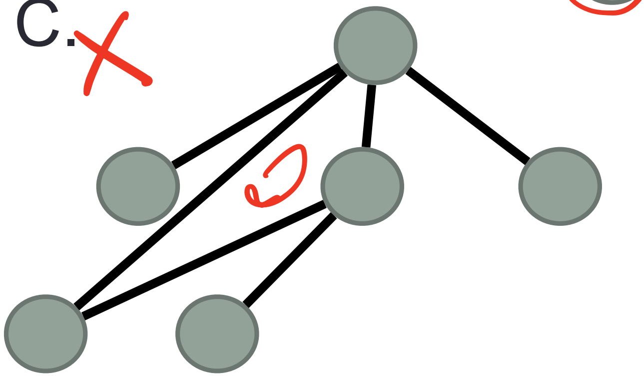
[✓ A.]



[✓ B.]



C. ✗

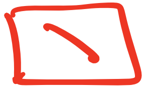


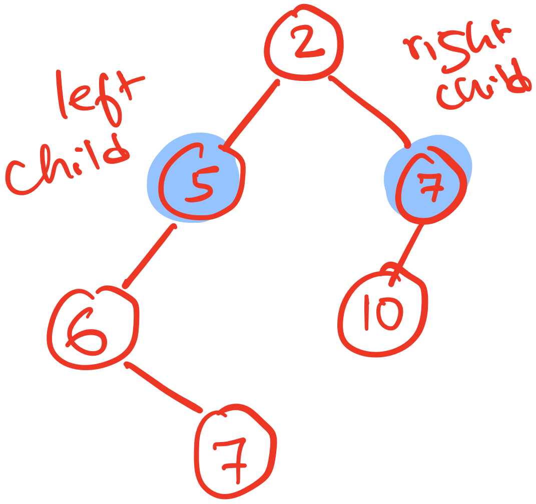
[✓ D.] A & B

E. All of A-C

Empty tree

root





Binary Tree

Binary Search Trees

- What are the operations supported?

Search!

- What are the running times of these operations?

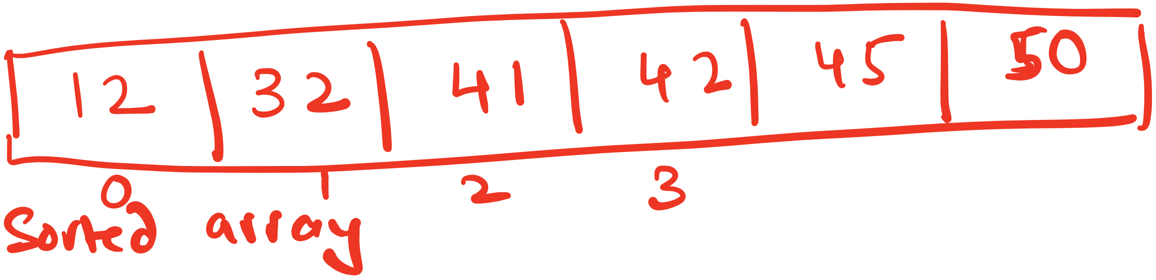
next lectures

- How do you implement the BST i.e. operations supported by it?

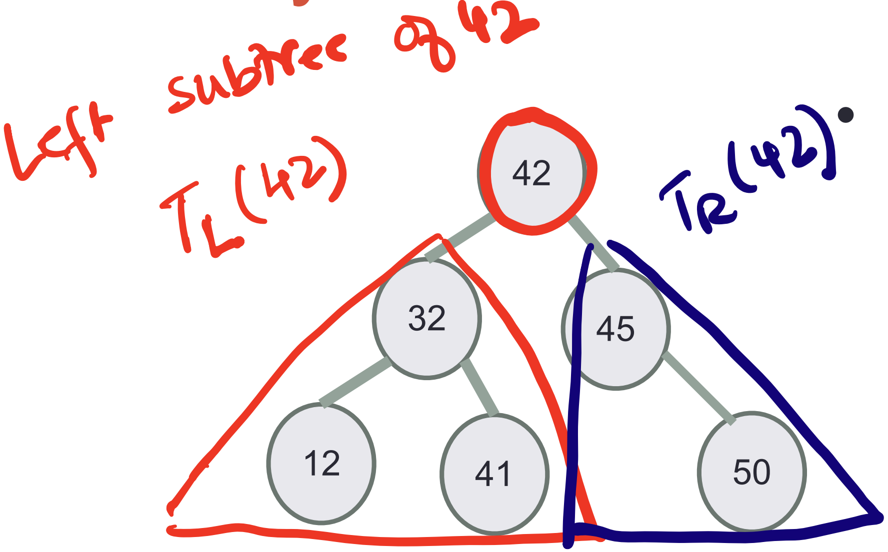
Operations supported by Sorted arrays and Binary Search Trees (BST)

Operations	
✓ Min	
✓ Max	
Successor (next larger key)	
Predecessor (next smaller key)	
✓ Search (Binary search)	
✓ Insert (not fast)	
✓ Delete (not fast)	
Print elements in order	

Example keys: 42, 32, 45, 12, 41, 50



Binary Search Tree – What is it?



- Each node:
  - stores a key (k)
  - has a pointer to left child, right child and parent (optional)
  - Satisfies the Search Tree Property

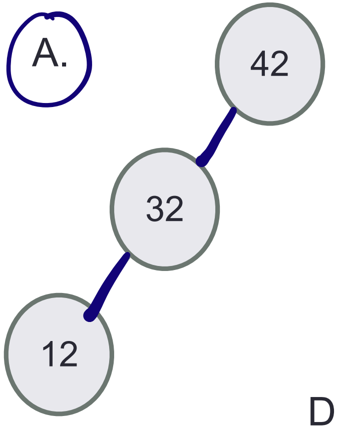
For any node,
Keys in node's left subtree < Node's key
Node's key < Keys in node's right subtree

$$T_L(k) < k < T_R(k)$$

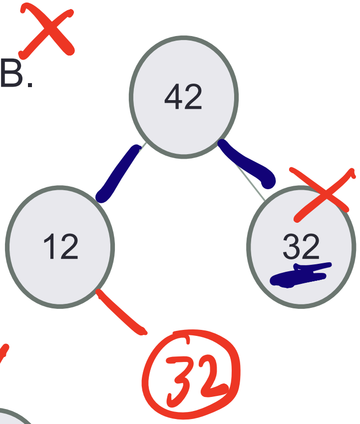
Do the keys have to be integers?

Which of the following is/are a binary search tree?

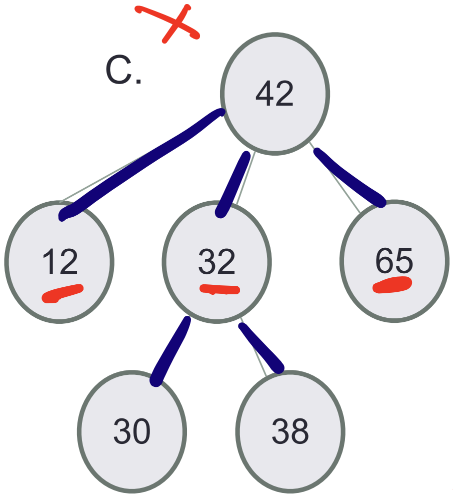
A.

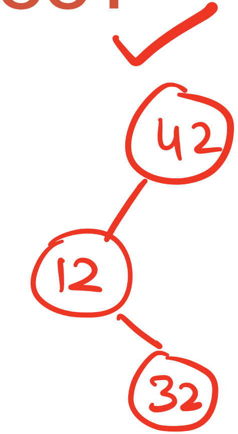


B.

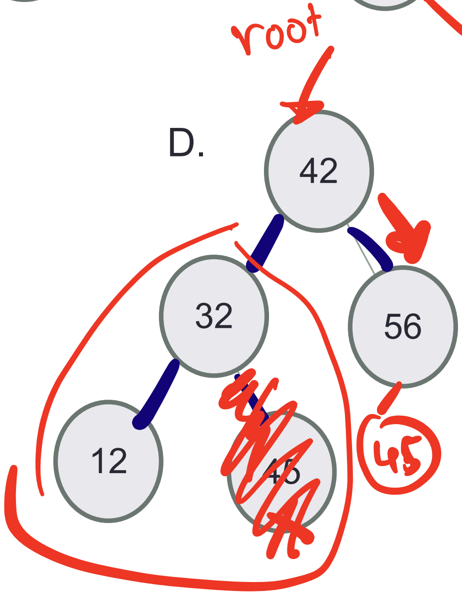


C.





D.

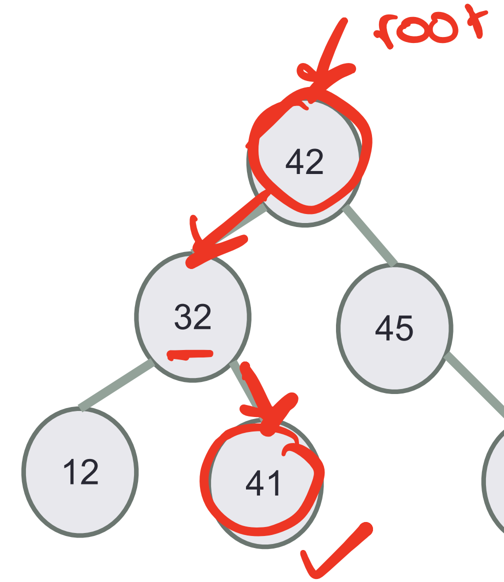


Insert 45 into tree D

C is not a binary tree

E. More than one of these

BSTs allow efficient search!



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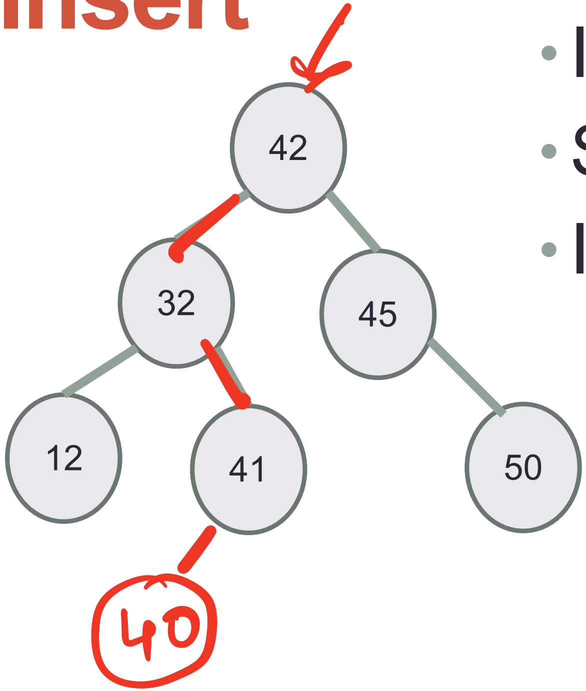
- Start at the root;
- Trace down a path by comparing **k** with the key of the current node x:
    - If the keys are equal: we have found the key
    - If **k** < key[x] search in the left subtree of x
    - If **k** > key[x] search in the right subtree of x

12 32 41 42 45 50

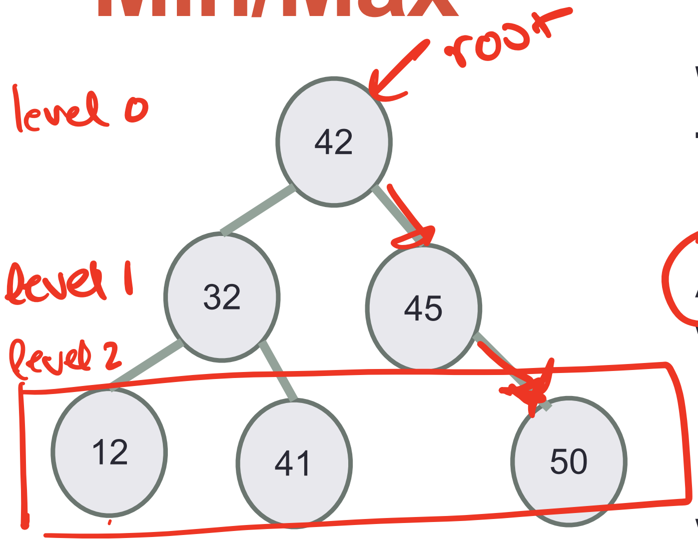
Search for 41, then search for 53

Insert



- Insert 40
- Search for the key
- Insert at the spot you expected to find it

Min/Max



Which of the following described the algorithm to find the maximum value in the BST?

[✓ A.] Follow right child pointers from the root, until a node with no right child is encountered, return that node's key

(min)

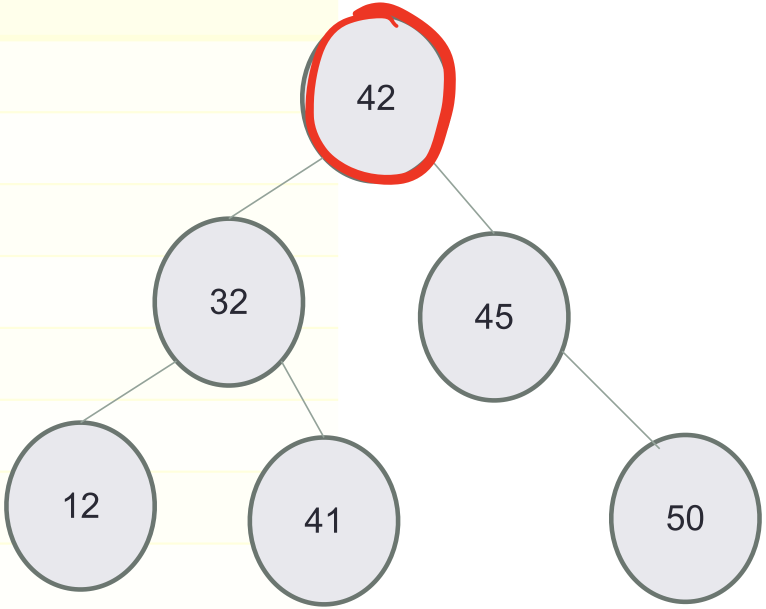
B. Follow left child pointers from the root, until a node with no left child is encountered, return that node's key

C. Traverse to the last level in the tree and traverse the tree left to right, return the key of the last node in the last level.

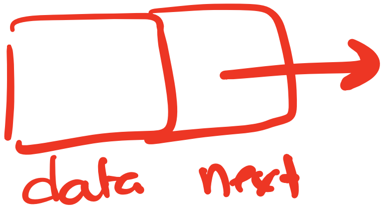
Description is not precise enough to be an algorithm
Need to define the notion of "level" & how to traverse a level from left to right.
Finally, even if we could carry out these steps, algo doesn't work for any BST. Look for a counterexample.

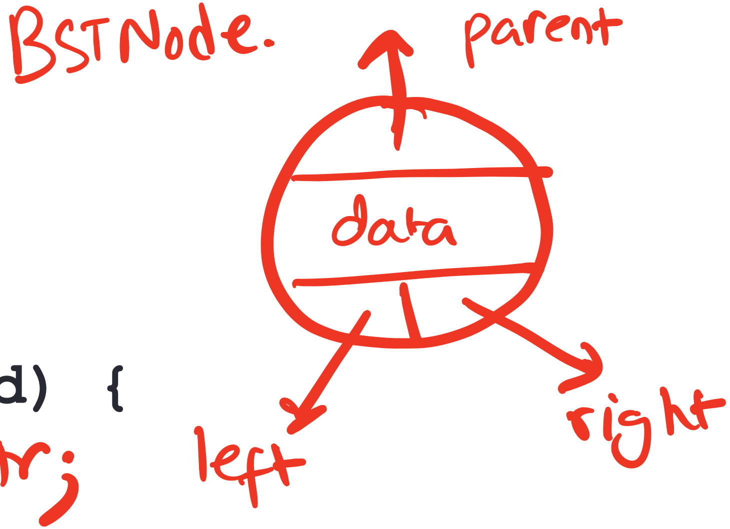
Define the BST ADT

Operations
Search
Insert
Min
Max
Successor
Predecessor
Delete
Print elements in order



```
class BSTNode {

public:
   BSTNode* left;
   BSTNode* right;
   BSTNode* parent;
   int const data;

   BSTNode( ~~const~~ int d ) : data(d) {
     left = right = parent = ;nullptr;
   }
};
```



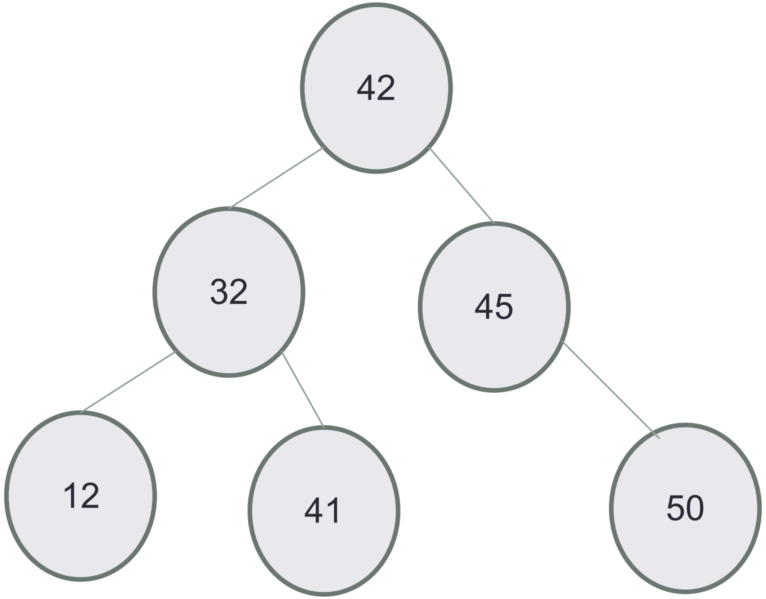


Traversing down the tree

- Suppose n is a pointer to the root. What is the output of the following code:

```
n = n->left;
n = n->right;
cout<<n->data<<endl;
```

A. 42
B. 32
C. 12
D. 41
E. Segfault



Traversing up the tree

- Suppose n is a pointer to the node with value 50.
- What is the output of the following code:

```
n = n->parent;
n = n->parent;
n = n->left;
cout<<n->data<<endl;
```

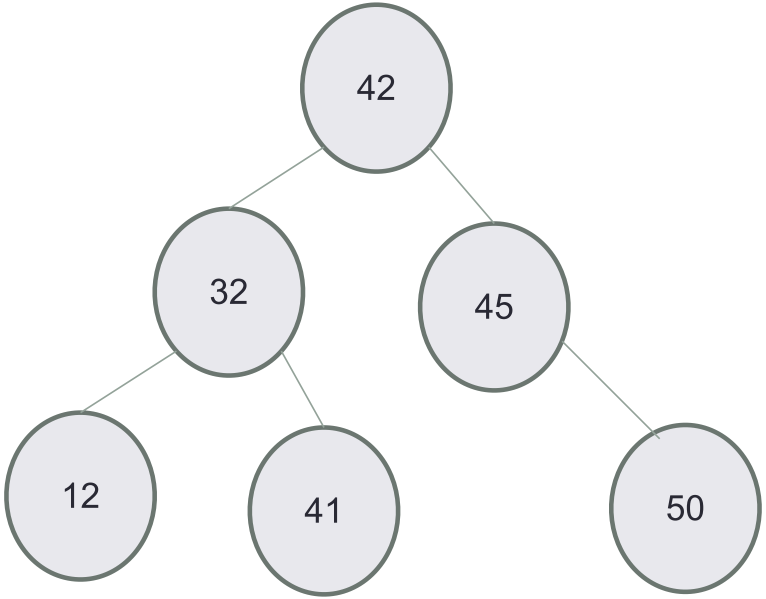
A. 42

B. 32

C. 12

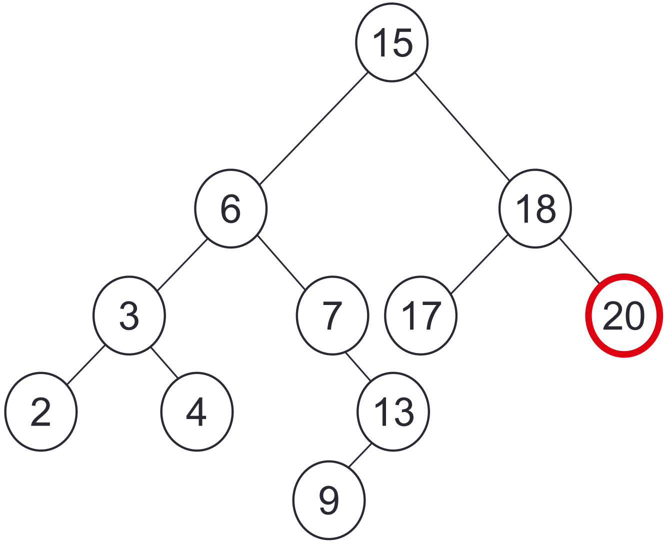
D. 45

E. Segfault



Max: find the maximum key value in a BST

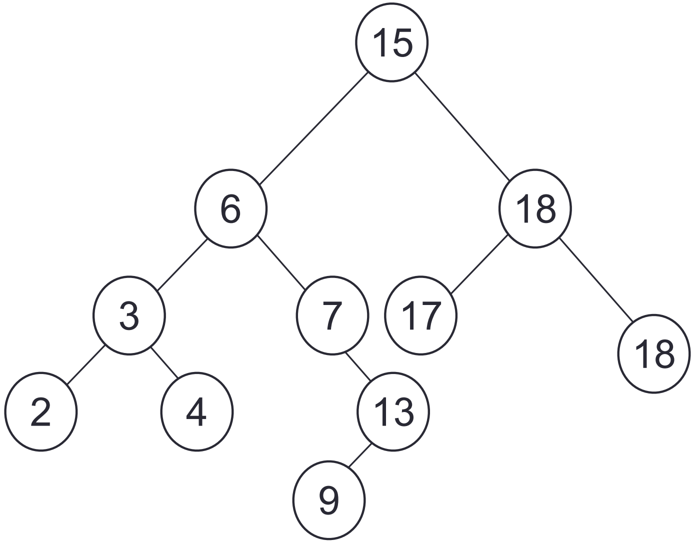
Alg: `int BST::max()`



Maximum = 20

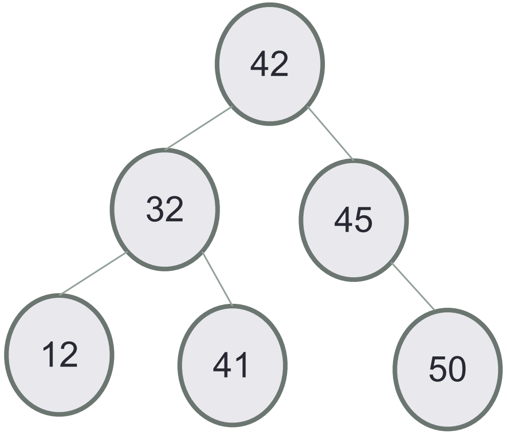
Min: find the minimum key value in a BST

```
Alg: int BST::min(){

 Start at the root.
 Follow __________ child
 pointers from the root, until
 a node with no left child is
 encountered.
 Return the key of that node

 }
```



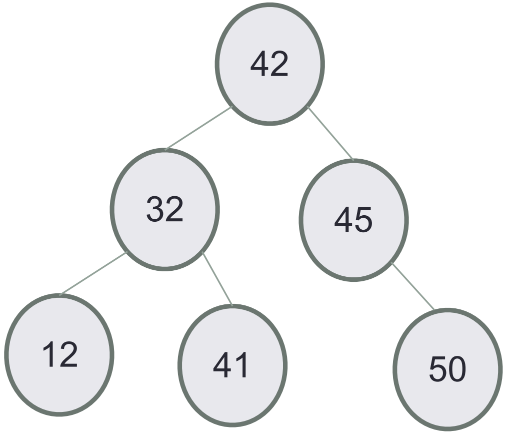
Min = ?

In order traversal: print elements in sorted order



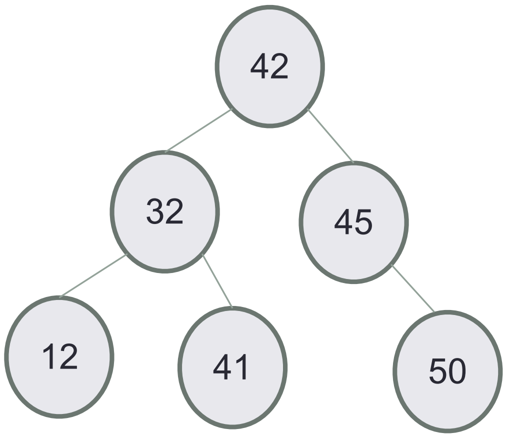
```
Algorithm Inorder(tree)
   1. Traverse the left subtree, i.e., call Inorder(left-subtree)
   2. Visit the root.
   3. Traverse the right subtree, i.e., call Inorder(right-subtree)
```

Pre-order traversal: nice way to linearize your tree!



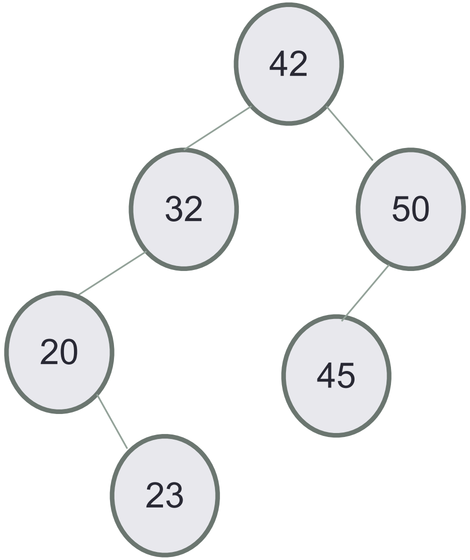
```
Algorithm Preorder(tree)
    1. Visit the root.
    2. Traverse the left subtree, i.e., call Preorder(left-subtree)
    3. Traverse the right subtree, i.e., call Preorder(right-subtree)
```

Post-order traversal: use in recursive destructors!



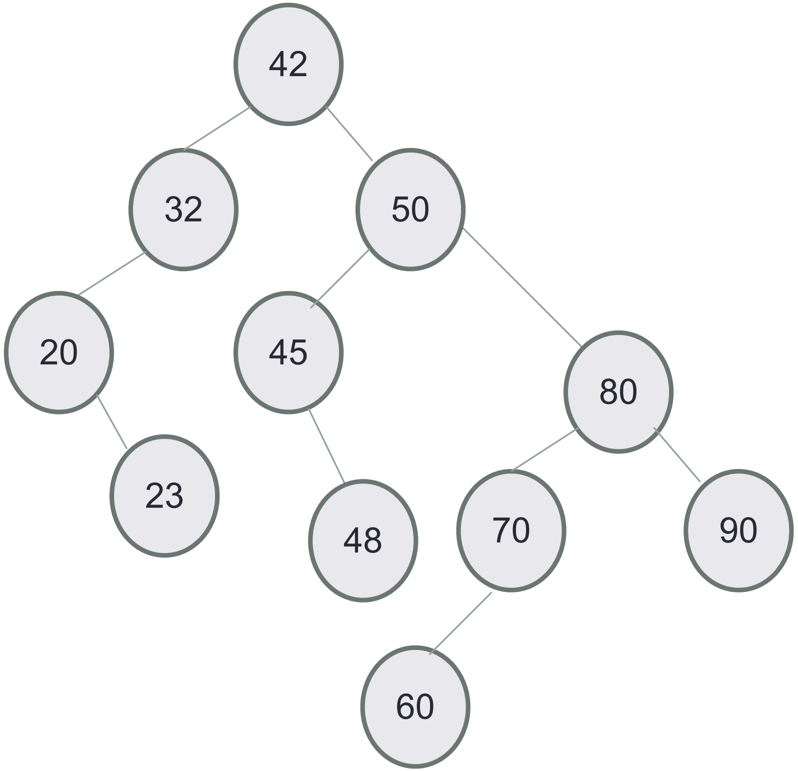
```
Algorithm Postorder(tree)
   1. Traverse the left subtree, i.e., call Postorder(left-subtree)
   2. Traverse the right subtree, i.e., call Postorder(right-subtree)
   3. Visit the root.
```

Predecessor: Next smallest element



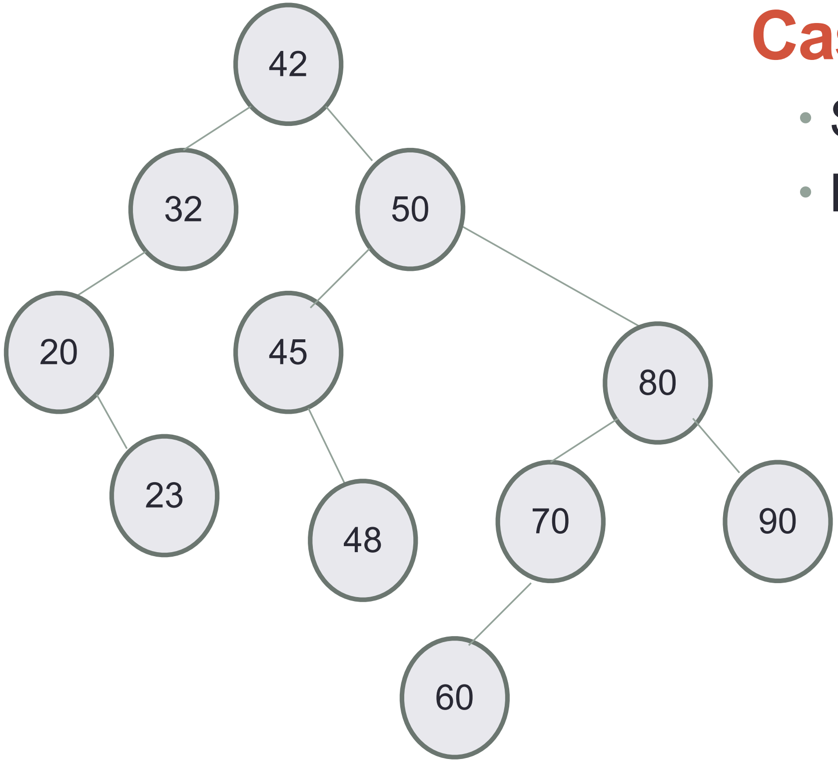
- What is the predecessor of 32?
- What is the predecessor of 45?

Successor: Next largest element



- What is the successor of 45?
- What is the successor of 50?
- What is the successor of 60?

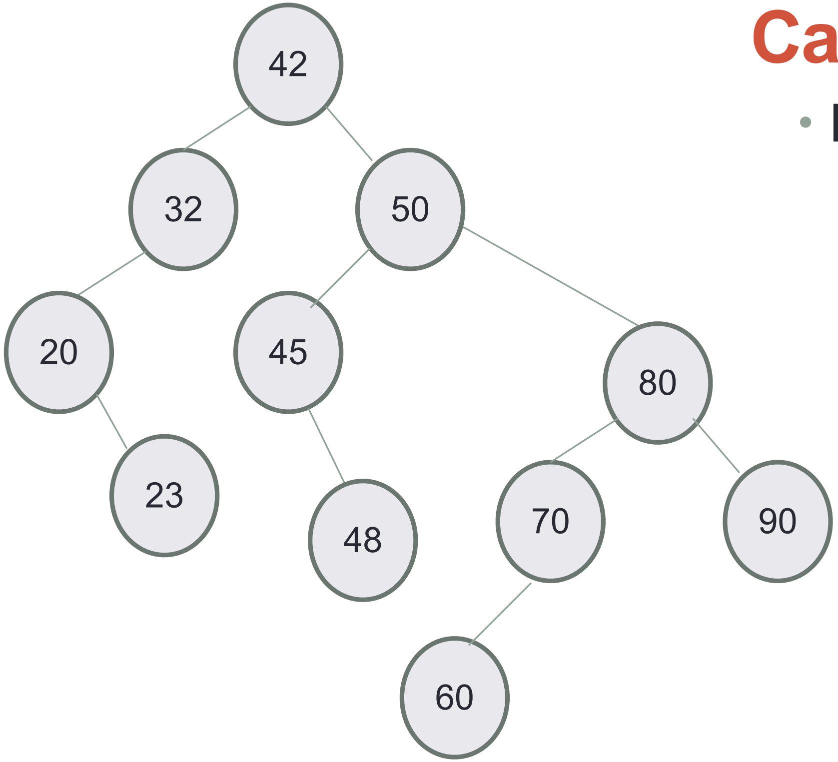
Delete: Case 1



Case 1: Node is a leaf node

- Set parent’s (left/right) child pointer to null
- Delete the node

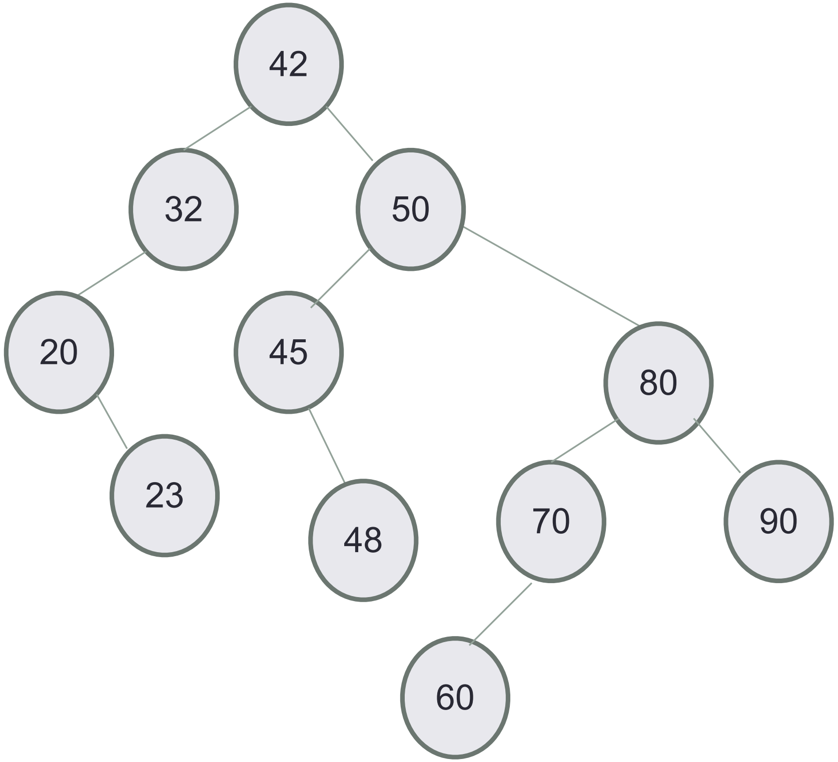
Delete: Case 2



Case 2 Node has only one child

- Replace the node by its only child

Delete: Case 3



Case 3 Node has two children

- Can we still replace the node by one of its children? Why or Why not?