

RUNNING TIME ANALYSIS

Problem Solving with Computers-II

C++

```
#include <iostream>
using namespace std;

int main(){
    cout<<"Hola Facebook!n";
    return 0;
}
```



Performance questions

- How efficient is a particular algorithm?
 - **CPU time usage (Running time complexity)**
 - Memory usage
 - Disk usage
 - Network usage
- Why does this matter?
 - Computers are getting faster, so is this really important?
 - Data sets are getting larger – does this impact running times?

How can we measure time efficiency of algorithms?

- One way is to measure the absolute running time
- Pros? Cons?

```
clock_t t;  
t = clock();  
  
//Code under test  
  
t = clock() - t;
```

Which implementation is significantly faster ?

A.

```
function F(n) {  
    if (n == 1) return 1  
    if (n == 2) return 1  
    return F(n-1) + F(n-2)  
}
```

B.

```
function F(n) {  
    Create an array fib[1..n]  
    fib[1] = 1  
    fib[2] = 1  
    for i = 3 to n:  
        fib[i] = fib[i-1] + fib[i-2]  
    return fib[n]  
}
```

C. *Both are almost equally fast*

A better question: How does the running time grow as a function of input size

```
function F(n) {  
    if (n == 1) return 1  
    if (n == 2) return 1  
    return F(n-1) + F(n-2)  
}
```

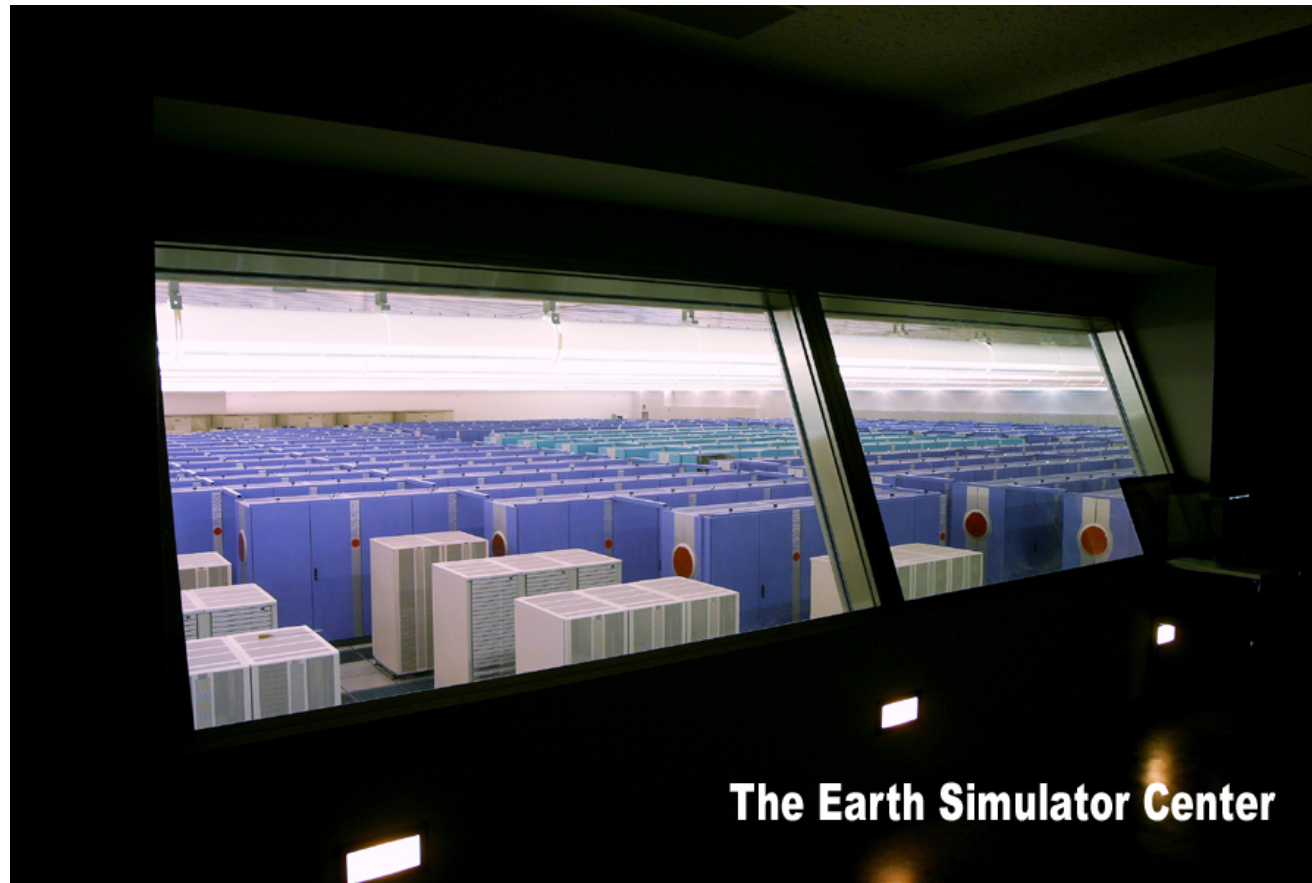
```
function F(n) {  
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    fib[1] = 1  
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    for i = 3 to n:  
        fib[i] = fib[i-1] + fib[i-2]  
    return fib[n]  
}
```

The “right” question is: How does the running time grow?

E.g. How long does it take to compute $F(200)$?

....let's say on....

NEC Earth Simulator



Can perform up to 40 trillion operations per second.

The running time of the recursive implementation

The Earth simulator needs 2^{92} seconds for F_{200} .

Time in seconds

2^{10}

2^{20}

2^{30}

2^{40}

2^{70}

Interpretation

17 minutes

12 days

32 years

cave paintings

The big bang!

```
function F(n) {  
    if (n == 1) return 1  
    if (n == 2) return 1  
    return F(n-1) + F(n-2)  
}
```

Let's try calculating F_{200}
using the iterative
algorithm on my laptop.....

Goals for measuring time efficiency

- **Focus on the impact of the algorithm:**

Simplify the analysis of running time by ignoring “details” which may be an artifact of the underlying implementation:

- E.g., $1000001 \approx 1000000$
- Similarly, $3n^2 \approx n^2$

- **Focus on trends as input size increases (asymptotic behavior):**

How does the running time of an algorithm increase with the size of the input in the limit (for large input sizes)

Counting steps (instead of absolute time)

- Every computer can do some primitive operations in constant time:
 - Data movement (assignment)
 - Control statements (branch, function call, return)
 - Arithmetic and logical operations
- By inspecting the pseudo-code, we can count the number of primitive operations executed by an algorithm

Running Time Complexity

Start by counting the primitive operations

```
/* N is the length of the array*/  
int sumArray(int arr[], int N)  
{  
    int result=0;  
    for(int i=0; i < N; i++)  
        result+=arr[i];  
    return result;  
}
```

Big-O notation

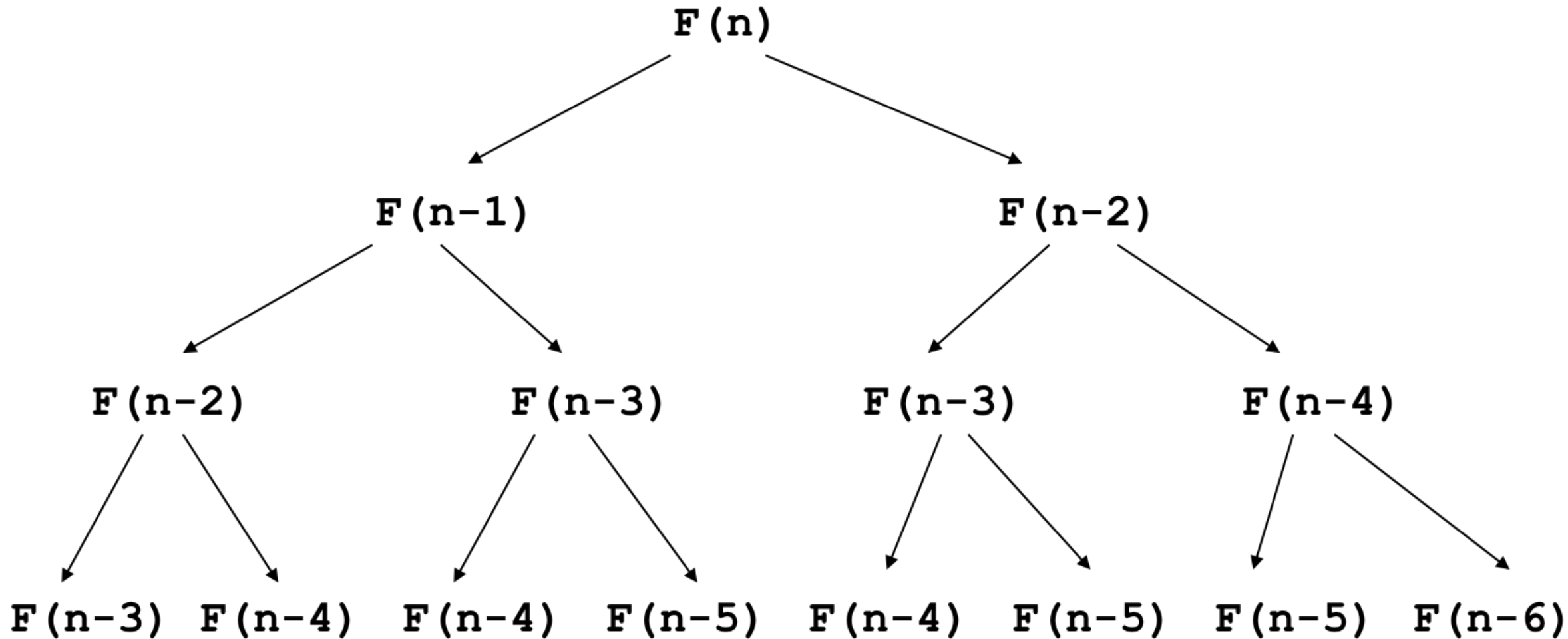
N	Steps = 5*N + 3
1	8
10	53
1000	5003
100000	500003
10000000	50000003

- Simplification 1: Count steps instead of absolute time
- Simplification 2: Ignore lower order terms
 - Does the constant 3 matter as N gets large?
- Simplification 3: Ignore constant coefficients in the leading term ($5*N$) simplified to N

After the simplifications,

The number of steps grows linearly in N
Running Time = $O(N)$ pronounced “Big-Oh of N”

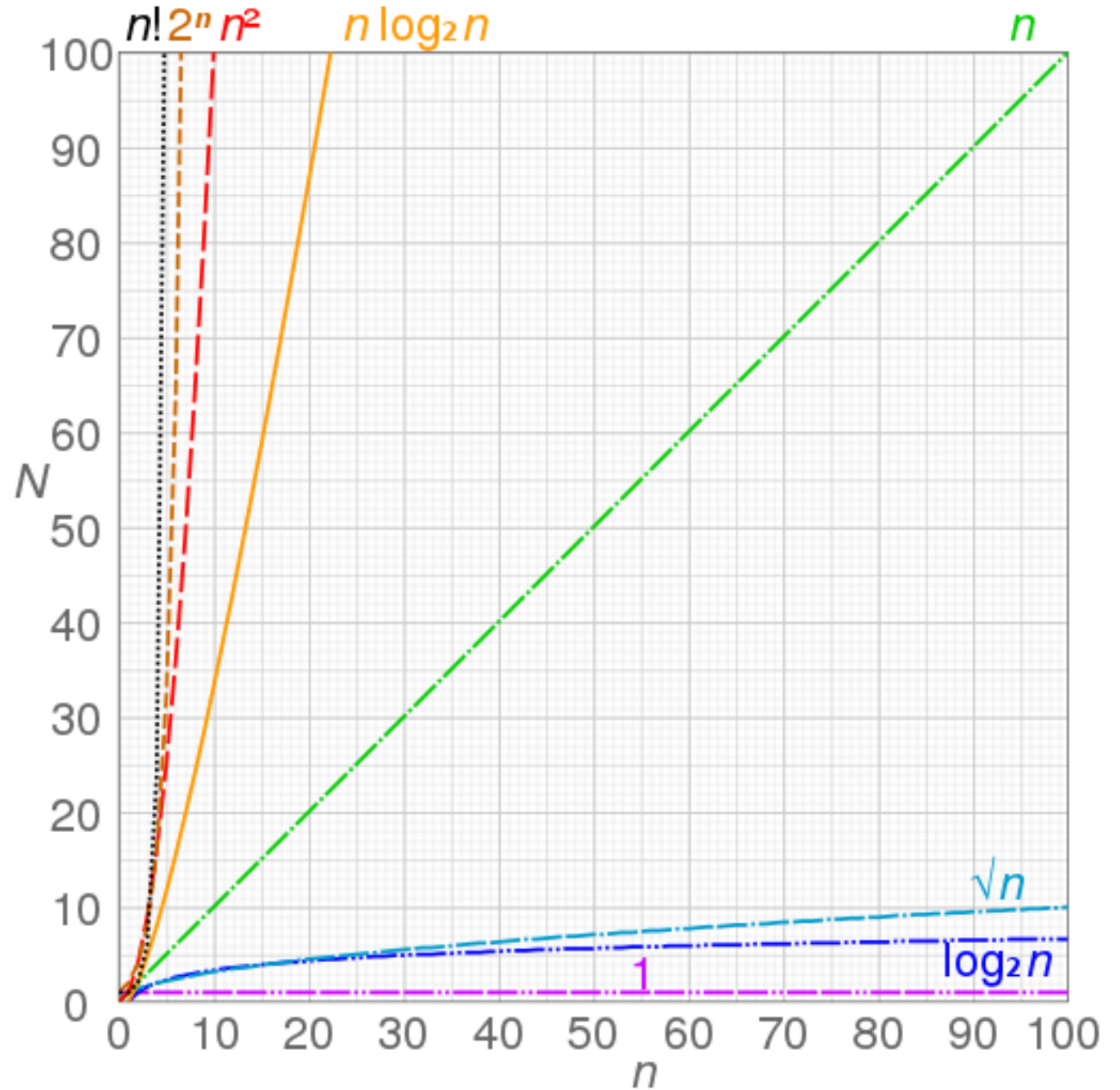
What takes so long? Let's unravel the recursion...



The same subproblems get solved over and over again!

Orders of growth

- We are interested in how algorithm running time scales with input size
- Big-Oh notation allows us to express that by ignoring the details
- 20n hours v. n^2 microseconds:
 - *which has a higher order of growth?*
 - *Which one is better?*



Big-O notation lets us focus on the big picture

Recall our goals:

- Focus on the impact of the algorithm
- Focus on asymptotic behavior (running time as N gets large)

Count the number of steps in your algorithm: $3 + 5 \cdot N$

Drop the constant additive term : $5 \cdot N$

Drop the constant multiplicative term : N

Running time grows linearly with the input size

Express the count using **O-notation**

Time complexity = $O(N)$

Given the step counts for different algorithms, express the running time complexity using Big-O

1. 10000000

2. $3*N$

3. $6*N-2$

4. $15*N + 44$

5. $50*N*\log N$

6. N^2

7. N^2-6N+9

8. $3N^2+4*\log(N)+1000$

For polynomials, use only leading term, ignore coefficients: linear, quadratic

Common sense rules of Big-O

1. Multiplicative constants can be omitted: $14n^2$ becomes n^2 .
2. n^a dominates n^b if $a > b$: for instance, n^2 dominates n .
3. Any exponential dominates any polynomial: 3^n dominates n^5 (it even dominates 2^n).

What is the Big O of sumArray2

- A. $O(N^2)$
- B. $O(N)$
- C. $O(N/2)$
- D. $O(\log N)$
- E. None of the array

```
/* N is the length of the array*/  
int sumArray2(int arr[], int N)  
{  
    int result=0;  
    for(int i=0; i < N; i=i+2)  
        result+=arr[i];  
    return result;  
}
```

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- C. $O(N/2)$
- D. $O(\log N)$
- E. None of the array

```
/* N is the length of the array*/  
int sumArray2(int arr[], int N)  
{  
    int result=0;  
    for(int i=1; i < N; i=i*2)  
        result+=arr[i];  
    return result;  
}
```


How is PA01 going?

- A. Done
- B. On track to finish
- C. Having trouble designing my classes
- D. Stuck and struggling
- E. Haven't started

Next time

- Running time analysis of Binary Search Trees

References:

<https://cseweb.ucsd.edu/classes/wi10/cse91/resources/algorithms.ppt>

<http://algorithmics.lsi.upc.edu/docs/Dasgupta-Papadimitriou-Vazirani.pdf>