RUNNING TIME ANALYSIS

Problem Solving with Computers-II

GitHub

Performance questions

- How efficient is a particular algorithm?
  - **CPU time usage (Running time complexity)**
  - Memory usage
  - Disk usage
  - Network usage

- Why does this matter?
  - Computers are getting faster, so is this really important?
  - Data sets are getting larger – does this impact running times?

How can we measure time efficiency of algorithms?

- One way is to measure the absolute running time
- Pros? Cons?

Cons: Time to run the algo for a specific input / input size
Doesn't tell us how the running time scales with the input size

* Tied to the performance of hardware
* Wait for the program to complete

```
#include <time>
clock_t t;
t = clock();
//Code under test
t = clock() - t;
```

number ticks of your computer clock

Algorithm

absolute time of running my code under test

(ticks)/CLOCKS_PER_SEC

Which implementation is significantly faster ?

A.

```
function F(n){
     if(n == 1) return 1
     if(n == 2) return 1
return F(n-1) + F(n-2)
}
```

B.

```
function F(n){
 Create an array fib[1..n]
 fib[1] = 1
 fib[2] = 1
 for i = 3 to n:
    fib[i] = fib[i-1] + fib[i-2]
 return fib[n]
}
```

C. *Both are almost equally fast*

Fibonacci F(n): 1 1 2 3 5 8 F(n-1) + F(n-2)

n: 1 2 3 4 5 6 ....n

A better question: How does the running time grow as a function of input size

```
function F(n){
    if(n == 1) return 1
    if(n == 2) return 1
return F(n-1) + F(n-2)
}
```

```
function F(n){
 Create an array fib[1..n]
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 for i = 3 to n:
    fib[i] = fib[i-1] + fib[i-2]
 return fib[n]
}
```
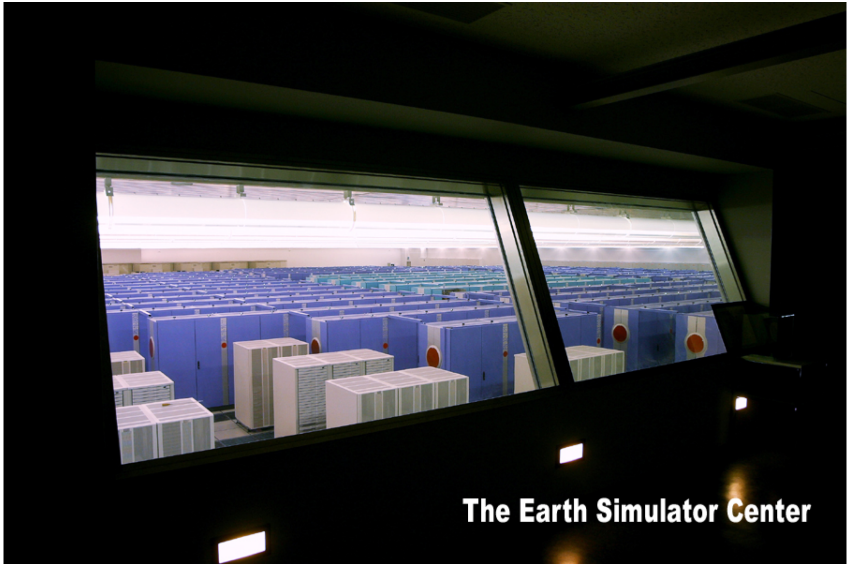

The “right” question is: How does the running time grow?

E.g. How long does it take to compute F(200)?

….let’s say on….

NEC Earth Simulator



Can perform up to 40 trillion operations per second.

The running time of the recursive implementation

The Earth simulator needs 2^{92} seconds for F_{200}.

Time in seconds	Interpretation
2^{10}	17 minutes
2^{20}	12 days
2^{30}	32 years
2^{40}	cave paintings
2^{70}	The big bang!

```
function F(n){
    if(n == 1) return 1
    if(n == 2) return 1
return F(n-1) + F(n-2)
}
```

Let's try calculating F_{200} using the iterative algorithm on my laptop…..

Goals for measuring time efficiency

- **Focus on the impact of the algorithm:**

Simplify the analysis of running time by ignoring “details” which may be an artifact of the underlying implementation:

  - E.g., 1000001 ≈ 1000000
  - Similarly, $3n^2 \approx n^2$

- **Focus on trends as input size increases (asymptotic behavior):**

How does the running time of an algorithm increases with the size of the input in the limit (for large input sizes)

Counting steps (instead of absolute time)

- Every computer can do some primitive operations in constant time:
  - Data movement (assignment) $x = 5$
  - Control statements (branch, function call, return)
  - Arithmetic and logical operations

- By inspecting the pseudo-code, we can count the number of primitive operations executed by an algorithm .

This is the first step for doing an analysis called Big-Oh

Running Time Complexity

Start by counting the primitive operations

```
/* N is the length of the array*/
int sumArray(int arr[], int N)
{
        int result=0;
        for(int i=0; i < N; i++)
                result+=arr[i];
        return result;
}
```

Statement	# of steps
int result=0	1
int i=0	1
i<N	1
i++	2
result+=arr[i]	3
return	1

Loop runs : N times

Total no. of steps

$= 1+1+N*(1+2+3)+1$

$= 3+6N$

Big-O notation

N	Steps = 5*N +3
1	8
10	53
1000	5003
100000	500003
10000000	50000003

- Simplification 1: Count steps instead of absolute time
- Simplification 2: Ignore lower order terms
  - Does the constant 3 matter as N gets large?
- Simplification 3: Ignore constant coefficients in the leading term (5*N) simplified to N

After the simplifications,

The number of steps grows linearly in N
Running Time = O(N) pronounced "Big-Oh of N"

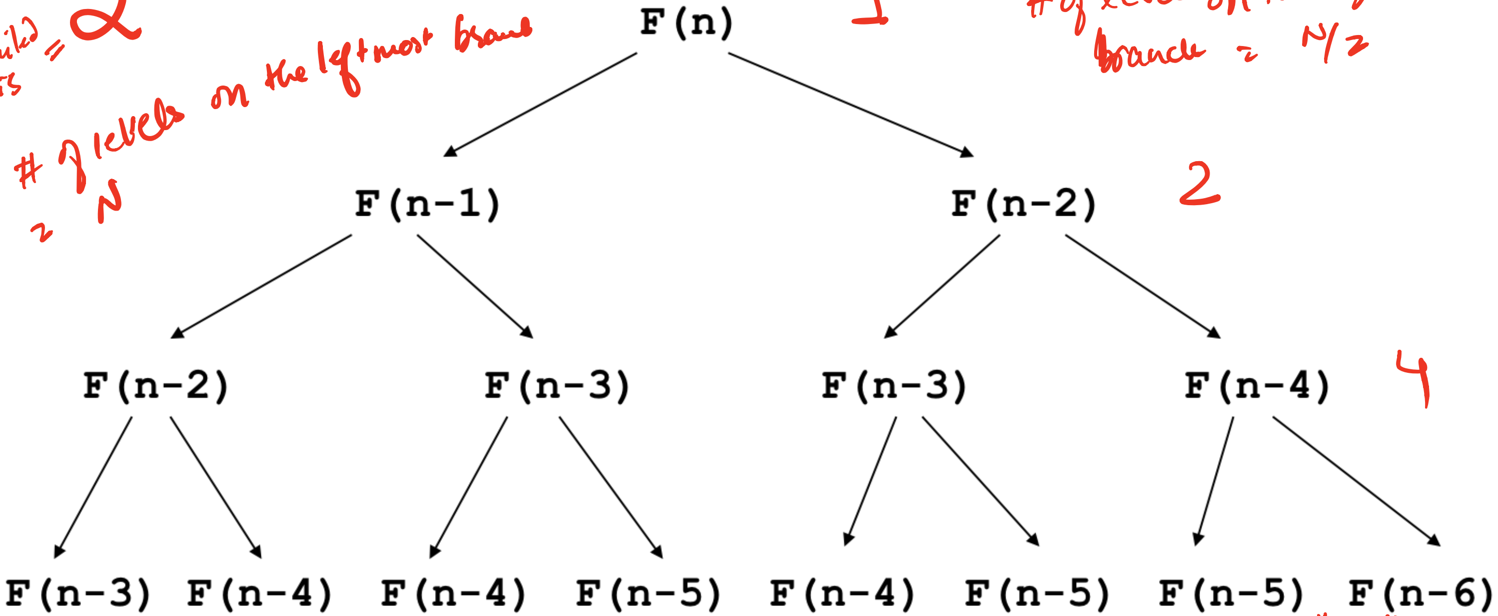
What takes so long? Let's unravel the recursion…

Running time = $O(2^N) \rightarrow$ exponential

# of steps from a more detailed analysis $= 2^{0.695N}$

# of levels on the leftmost branch $= N$

# of levels on the right most branch $= N/2$



1

2

4

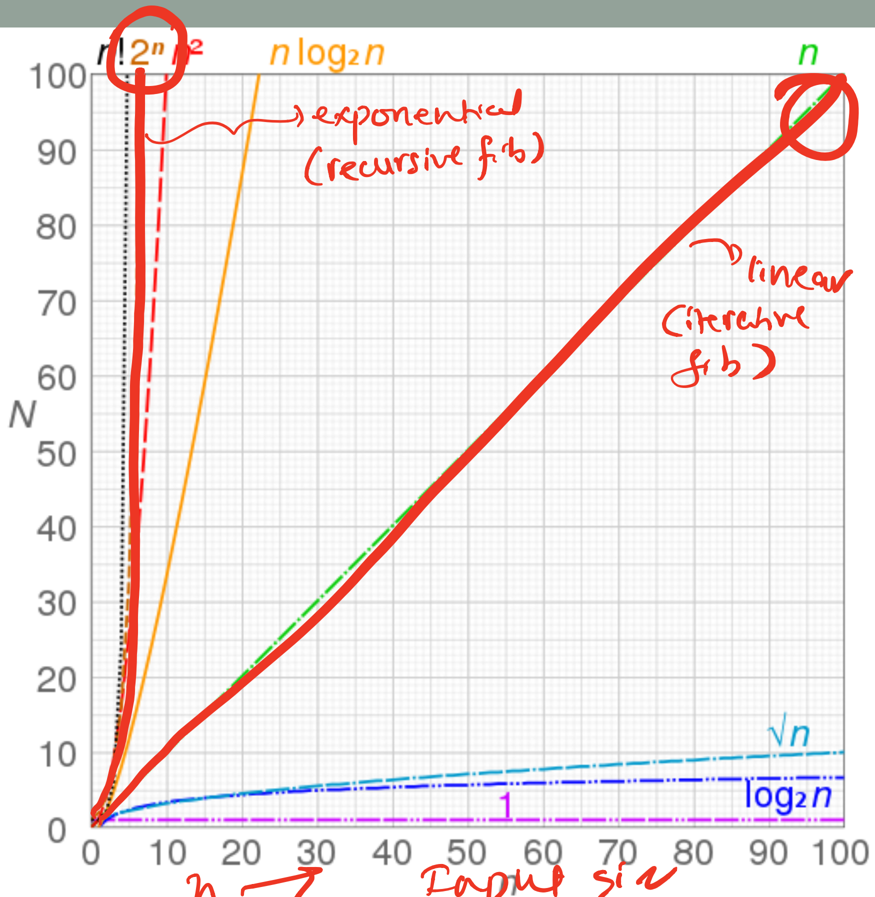
2^N

# of function calls at level $k = 2^k$. Total # of function calls is between $2^{N/2}$ and 2^N

The same subproblems get solved over and over again! $O(2^n)$

Orders of growth

- We are interested in how algorithm running time scales with input size
- Big-Oh notation allows us to express that by ignoring the details
- 20n hours v. n^2 microseconds:
  - *which has a higher order of growth?*
  - *Which one is better?*



Big-O notation lets us focus on the big picture

Recall our goals:

- **Focus on the impact of the algorithm**
- **Focus on asymptotic behavior (running time as N gets large)**

Count the number of steps in your algorithm: 3+ 5*N
Drop the constant additive term : 5*N
Drop the constant multiplicative term : N
Running time grows linearly with the input size
Express the count using **O-notation**
Time complexity = O(N)

Given the step counts for different algorithms, express the running time complexity using Big-O

Number of steps

1. 10000000 — $O(1)$
2. 3*N — $O(N)$
3. 6*N-2 — $O(N)$
4. 15*N + 44 — $O(N)$
5. 50*N*logN — $O(N \log N)$
6. N^2 — $O(N^2)$
7. N^2-6N+9 — $O(N^2)$
8. $3N^2+4*\log(N)+1000$ — $O(N^2)$

$2^N + N^{20} + N \log N$ — $O(2^N)$

For polynomials, use only leading term, ignore coefficients: linear, quadratic

Common sense rules of Big-O

1. Multiplicative constants can be omitted: $14n^2$ becomes n^2 .
2. n^a dominates n^b if a > b: for instance, n^2 dominates n.
3. Any exponential dominates any polynomial: 3^n dominates n^5 (it even dominates 2^n).

What is the Big O of sumArray2

A. $O(N^2)$
B. $O(N)$
C. $O(N/2)$
D. $O(\log N)$
E. None of the array

```
/* N is the length of the array*/
int sumArray2(int arr[], int N)
{
        int result=0;
        for(int i=0; i < N; i=i+2)
                result+=arr[i];
        return result;
}
```

c_1

c_2

$$c_1 + c_2 * \frac{N}{2}$$

What is the Big O of sumArray2

A. $O(N^2)$
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```
/* N is the length of the array*/
int sumArray2(int arr[], int N)
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        for(int i=1; i < N; i=i*2)
                result+=arr[i];
        return result;
}
```

c_1

c_2

Total number of steps = $c_1 + c_2 * \#$ of iterations of loop

The important step here is to get the number of times the loop runs as a function of N

To do that we can get a relationship between the iteration number k and the loop variable i

Iteration number	i
1	1
2	2
3	4
4	8
⋮	
k →	2^{k-1}

In general at iteration k
$i = 2^{k-1}$

The for loop stops when i is greater than or equal to N

$$2^{k-1} >= N$$

Solve for k

$$k > \log N + 1$$

Total No. of steps $= C_1 + C_2 * (\log N + 1)$

Apply the rules of Big-O

$$O(\log N)$$

Operations on sorted arrays

- Min :
- Max:
- Median:
- Successor:
- Predecessor:
- Search:
- Insert :
- Delete:

6	13	14	25	33	43	51	53	64	72	84	93	95	96	97
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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How is PA01 going?

A. Done
B. On track to finish
C. Having trouble designing my classes
D. Stuck and struggling
E. Haven’t started

- PA02 deadline this Thursday (04/18)at midnight

Next time

- Running time analysis of Binary Search Trees

References:
https://cseweb.ucsd.edu/classes/wi10/cse91/resources/algorithms.ppt
http://algorithmics.lsi.upc.edu/docs/Dasgupta-Papadimitriou-Vazirani.pdf