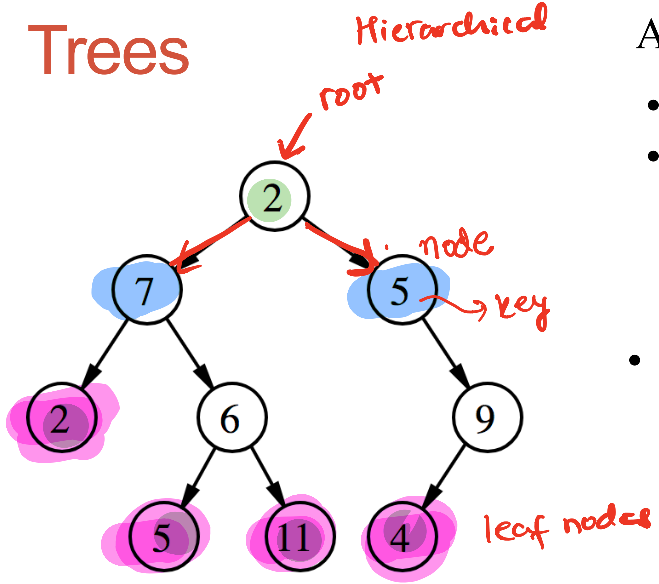
BINARY SEARCH TREES

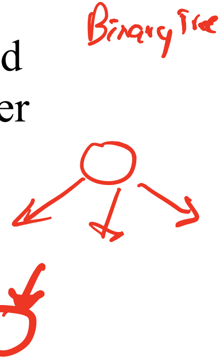
Problem Solving with Computers-II

Trees

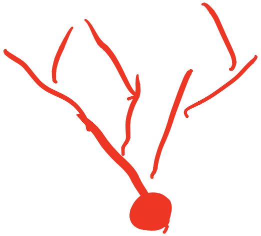


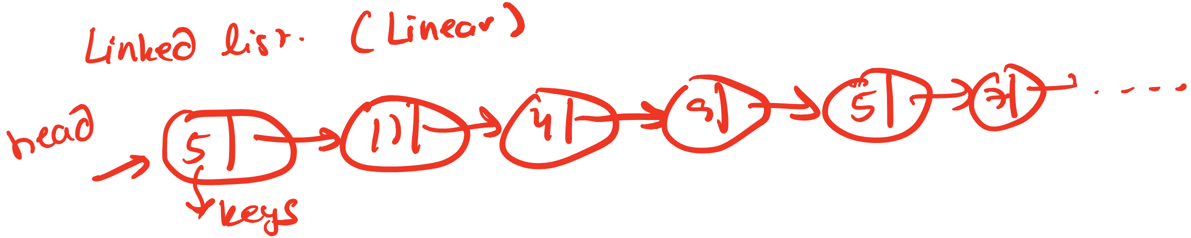
A tree has following general properties:

- One node is distinguished as a **root**;
- Every node (exclude a root) is connected by a directed edge *from* exactly one other node;
  A direction is: *parent -> children*
- *Leaf node: Node that has no children*



2 is the root node
7 & 5 are 2's children
2 is the parent of 7 & 5

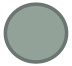




Which of the following is/are a tree?

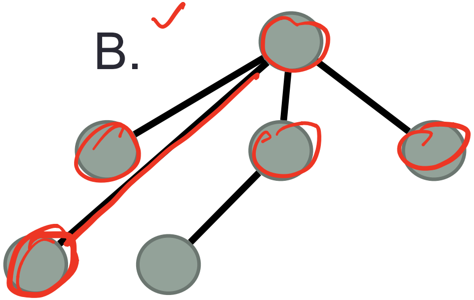
root
Empty tree

A.

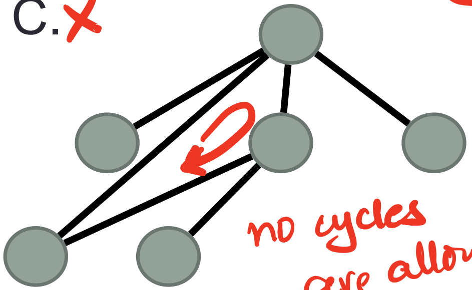


Single node tree

B.



C.



no cycles are allowed

D. A & B

E. All of A-C

Binary Search Trees (BST)

2	10	15	30

- What are the operations supported?

  All operations that are supported by array & linked list

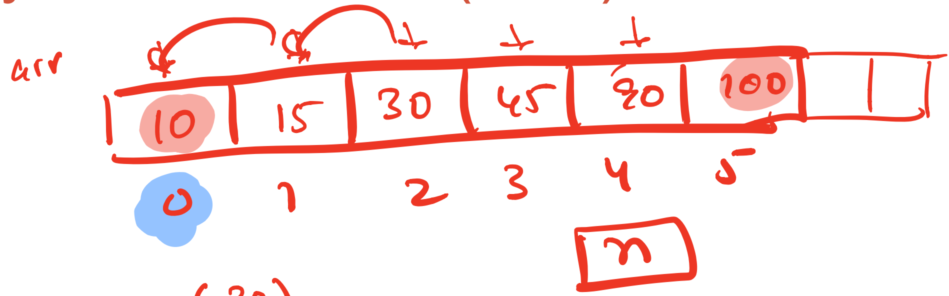
  sorted array + fast deletion & insertion.

- What are the running times of these operations?

- How do you implement the BST i.e. operations supported by it?

Sorted arrays vs Binary Search Trees (BST)

Operations
Min
Max
Successor - next largest key
Predecessor - next smallest key
Search
Insert
Delete
Print elements in order



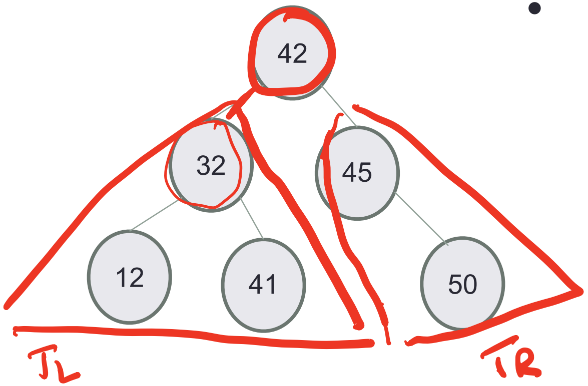
Successor (30)

linear search

special type of binary search

Insert, Delete } fast in BST

Binary Search Tree – What is it?



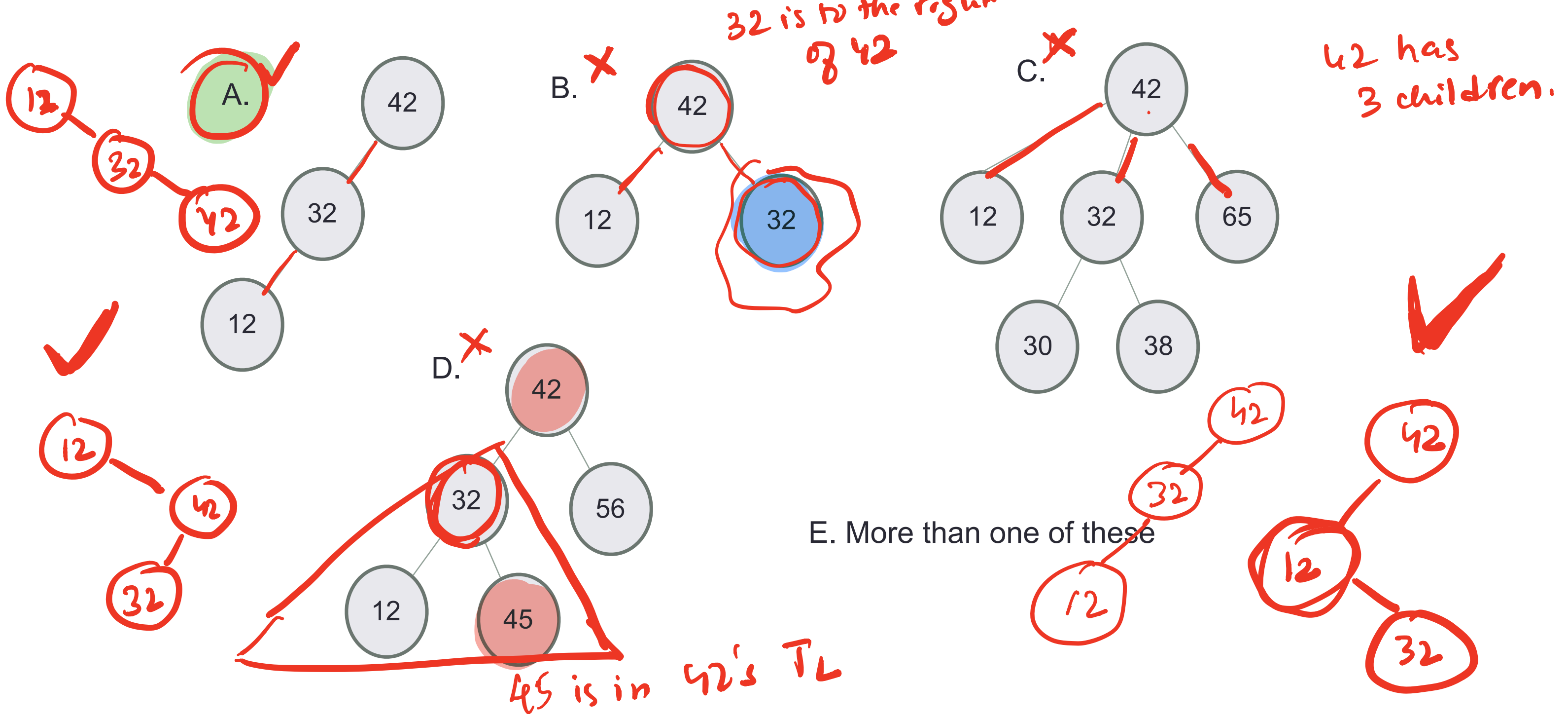
- Each node:
  - stores a key (k)
  - has a pointer to left child, right child and parent (optional)
  - Satisfies the Search Tree Property

For any node,
Keys in node's left subtree <= Node's key
Node's key < Keys in node's right subtree

$keys(T_L(x)) < x < keys(T_R(x))$

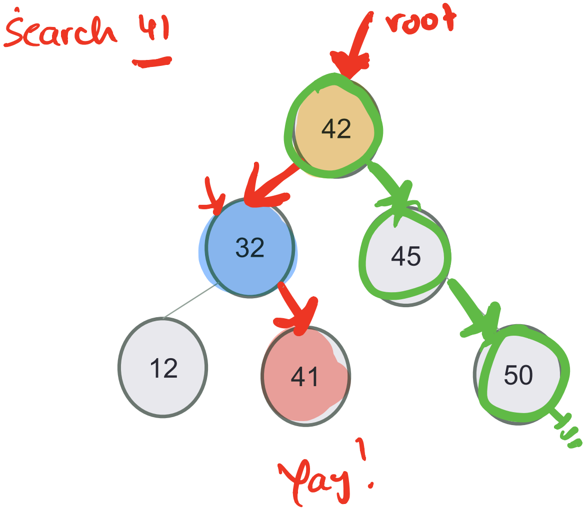
Do the keys have to be integers?

Which of the following is/are a binary search tree?



E. More than one of these

BSTs allow efficient search!



- Start at the root;
- Trace down a path by comparing **k** with the key of the current node x:
  - If the keys are equal: we have found the key
  - If **k** < key[x] search in the left subtree of x
  - If **k** > key[x] search in the right subtree of x

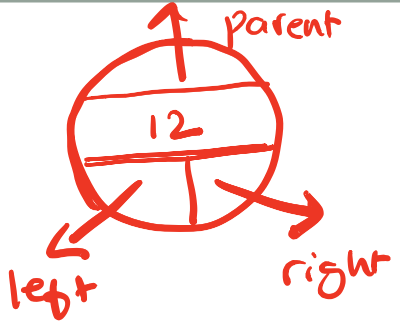
Search for 41, then search for 53

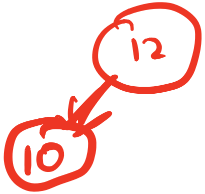
A node in a BST

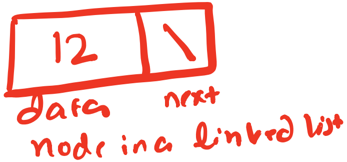
```
class BSTNode {

public:
   BSTNode* left;
   BSTNode* right;
   BSTNode* parent;
   int const data;

   BSTNode( const int & d ) : data(d) {
     left = right = parent = nullptr;
   }
};
```



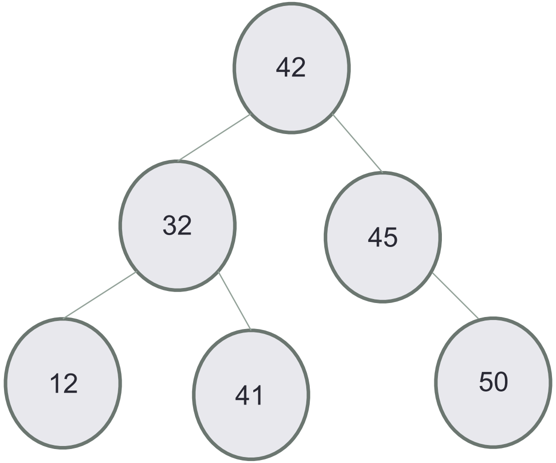





```
struct Node {
  int data;
  Node* next;
}
```

Define the BST ADT

Operations
Search
Insert
Min
Max
Successor
Predecessor
Delete
Print elements in order

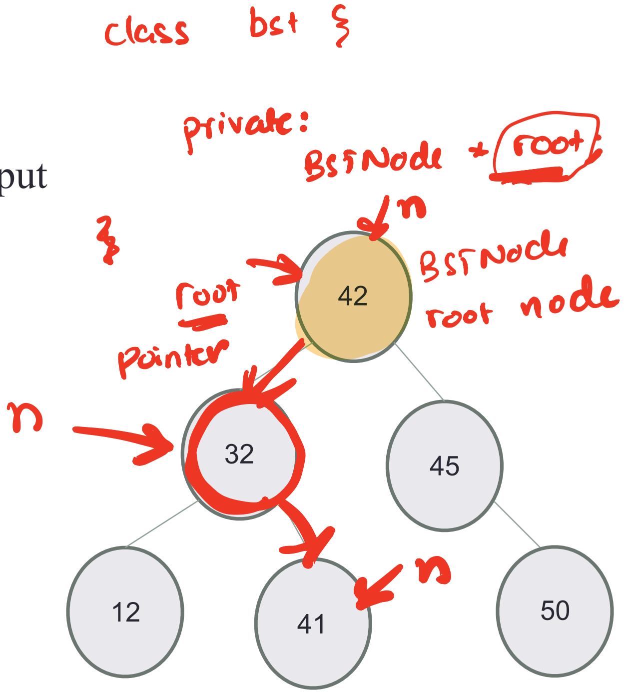


Traversing down the tree

- Suppose n is a pointer to the root. What is the output of the following code:

```
n = n->left;
n = n->right;
cout<<n->data<<endl;
```

A. 42
B. 32
C. 12
D. 41
E. Segfault



Traversing up the tree

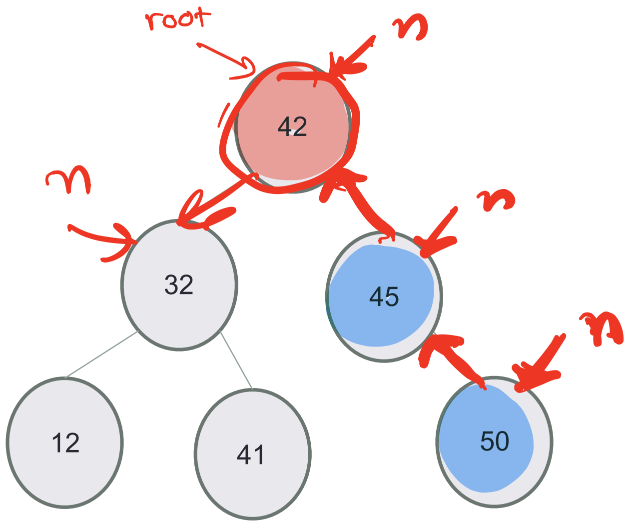
- Suppose n is a pointer to the node with value 50.
- What is the output of the following code:

```
n = n->parent;
n = n->parent;
n = n->left;
cout<<n->data<<endl;
```

A. 42
B. 32
C. 12
D. 45
E. Segfault

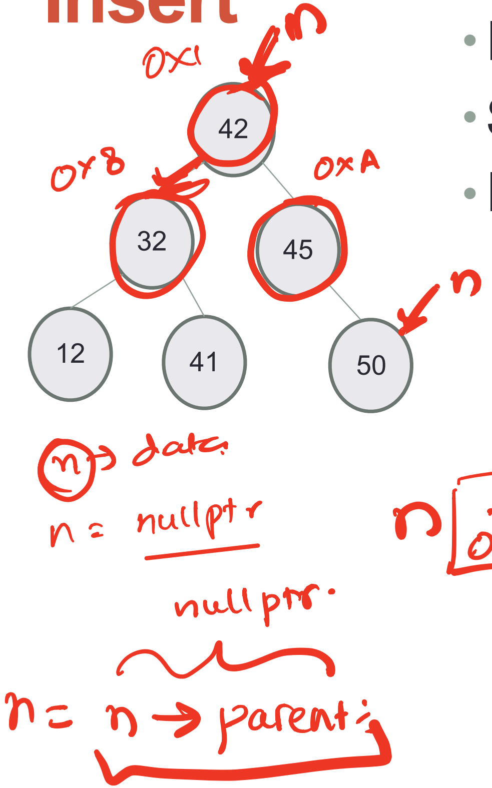
```
BSTNode *n = root;
while (n && n->right) {
    n = n->right;
}
cout << n->data;
```

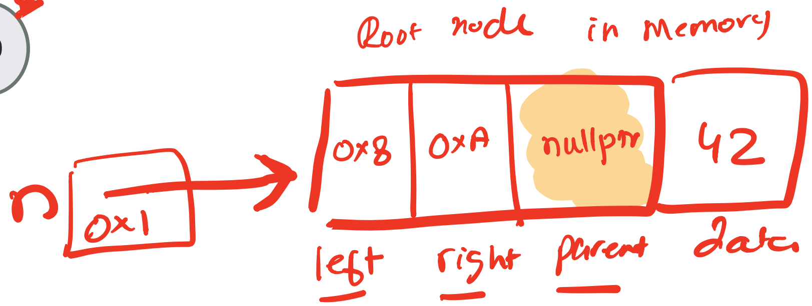
n->left = dereferencing. n.



Insert

- Insert 40
- Search for the key
- Insert at the spot you expected to find it



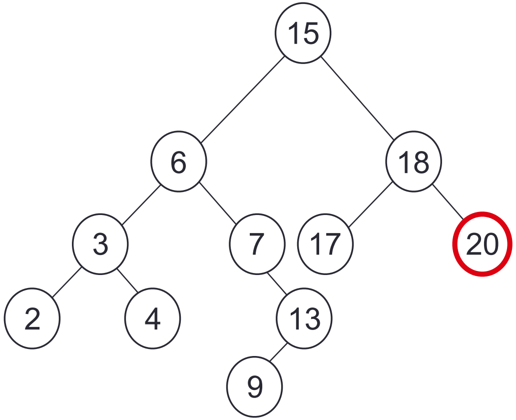


Max

Goal: find the maximum key value in a BST

Following right child pointers from the root, until a leaf node is encountered. The least node has the max value

Alg: `int BST::max()`



Maximum = 20

Min

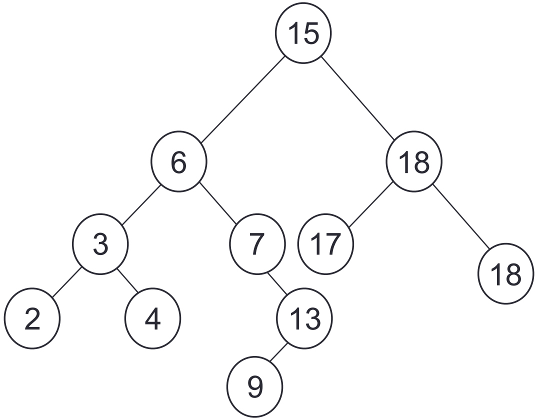
Goal: find the minimum key value in a BST

Start at the root.

Follow _________ child pointers from the root, until a leaf node is encountered

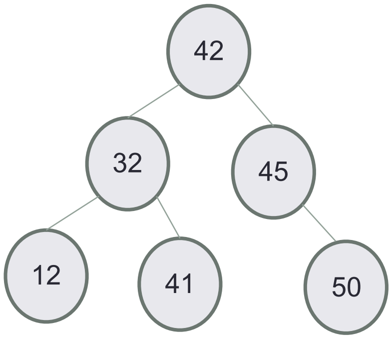
Leaf node has the min key value

Alg: `int BST::min()`



Min = ?

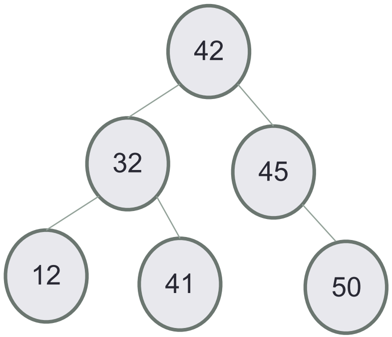
In order traversal: print elements in sorted order



Algorithm Inorder(tree)

1. Traverse the left subtree, i.e., call Inorder(left-subtree)
2. Visit the root.
3. Traverse the right subtree, i.e., call Inorder(right-subtree)

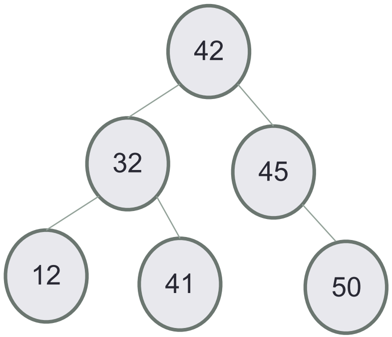
Pre-order traversal: nice way to linearize your tree!



Algorithm Preorder(tree)

1. Visit the root.
2. Traverse the left subtree, i.e., call Preorder(left-subtree)
3. Traverse the right subtree, i.e., call Preorder(right-subtree)

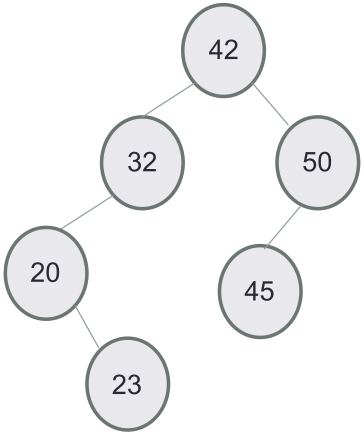
Post-order traversal: use in recursive destructors!



Algorithm Postorder(tree)

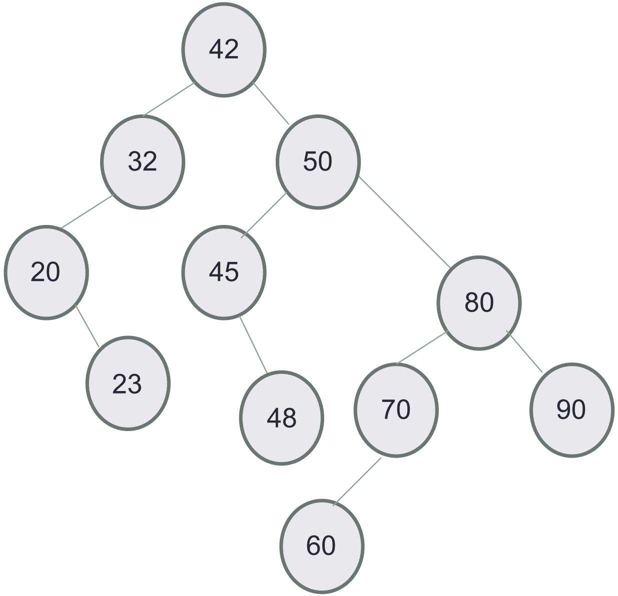
1. Traverse the left subtree, i.e., call Postorder(left-subtree)
2. Traverse the right subtree, i.e., call Postorder(right-subtree)
3. Visit the root.

Predecessor: Next smallest element



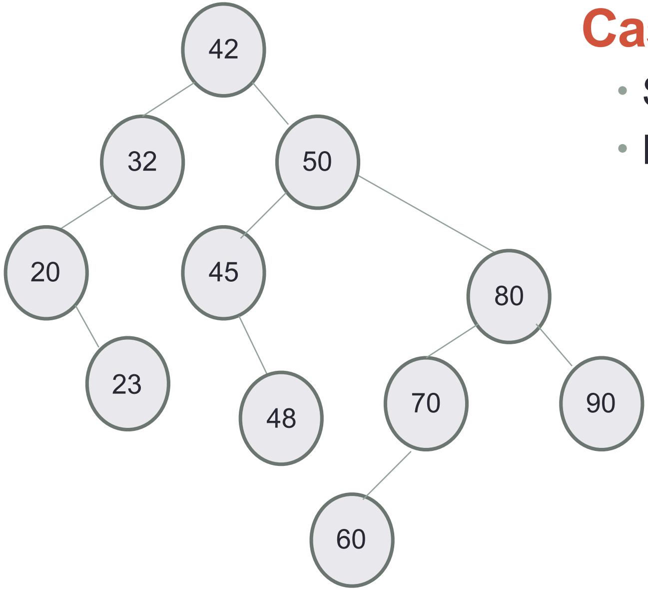
- What is the predecessor of 32?
- What is the predecessor of 45?

Successor: Next largest element



- What is the successor of 45?
- What is the successor of 50?
- What is the successor of 60?

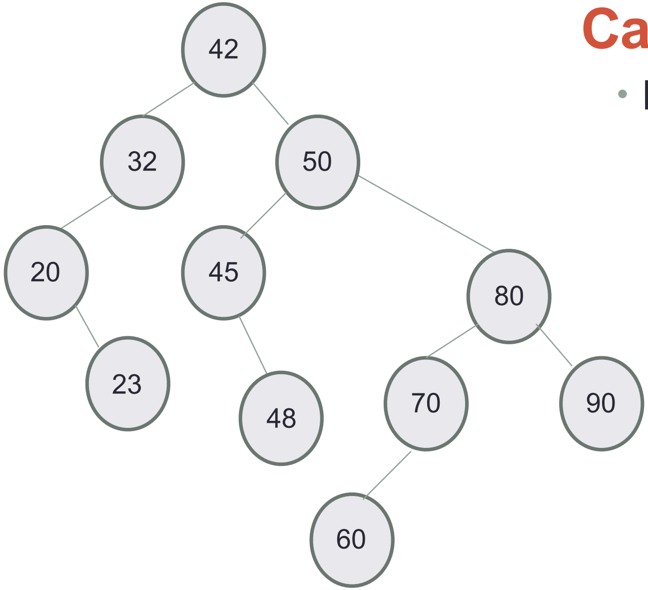
Delete: Case 1



Case 1: Node is a leaf node

- Set parent's (left/right) child pointer to null
- Delete the node

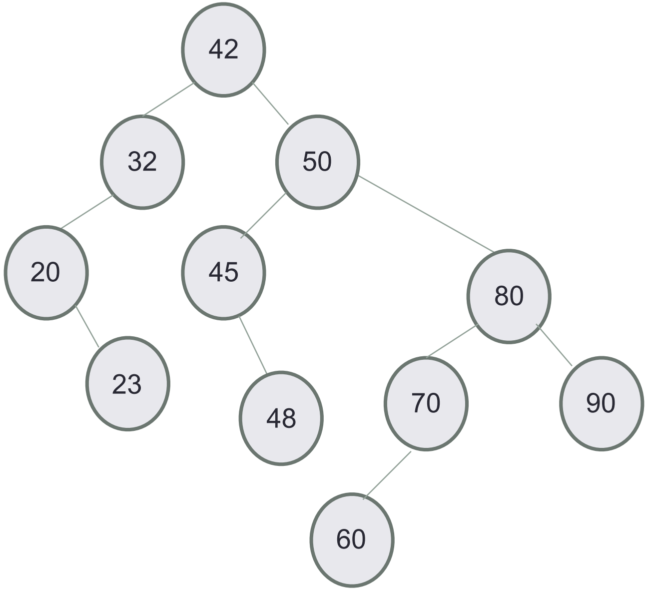
Delete: Case 2



Case 2 Node has only one child

- Replace the node by its only child

Delete: Case 3



Case 3 Node has two children

- Can we still replace the node by one of its children? Why or Why not?

Binary Search

- Binary search. Given `value` and sorted array `a[]`, find index `i` such that `a[i]` = `value`, or report that no such index exists.
- Invariant. Algorithm maintains `a[lo]` ≤ `value` ≤ `a[hi]`.

- Ex. Binary search for 33.

6	13	14	25	33	43	51	53	64	72	84	93	95	96	97
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
↑														↑
lo														hi