RUNNING TIME ANALYSIS

Problem Solving with Computers-II

Quiz 2 Today 6PM-9PM

Lab02 Due Wed

PA01 Released 5/12 (Wed)

Friday zybook Activities chapt 4.

Performance questions

- How efficient is a particular algorithm?
  - **CPU time usage (Running time complexity)**
  - Memory usage
  - Disk usage
  - Network usage
- Why does this matter?
  - Computers are getting faster, so is this really important?
  - Data sets are getting larger – does this impact running times?

How can we measure time efficiency of algorithms?

- One way is to measure the absolute running time
- Pros? Cons?
  - One data point, specific input
  - Long time to run for inefficient also.
  - Depends on hardware

#include <time>

```
clock_t t;
t = clock();

//Code under test

t = clock() - t;
```

Difference in time (number of ticks)

s = t/CLOCKS_PER_SEC;

Which implementation is significantly faster ?

$O(n)$ ✓

A.

```
function F(n){
     if(n == 1) return 1
     if(n == 2) return 1
return F(n-1) + F(n-2)
}
```

$T_R(n) = 2^{0.69n}$

$= O(2^n)$

B.

```
function F(n){
 Create an array fib[1..n]   1
 fib[1] = 1                  1
 fib[2] = 1                  1
 for i = 3 to n:
    fib[i] = fib[i-1] + fib[i-2]  4
 return fib[n]
}
```

n-3

$T_I(n) = 3 + (n-3) * 4$

$= 4n - 9$

$= O(N)$

C. *Both are almost equally fast*

F(n)	1	1	2	3	5	8	
n	1	2	3	4	5	6	7

A better question: How does the running time grow as a function of input size

```
function F(n){
     if(n == 1) return 1
     if(n == 2) return 1
return F(n-1) + F(n-2)
}
```

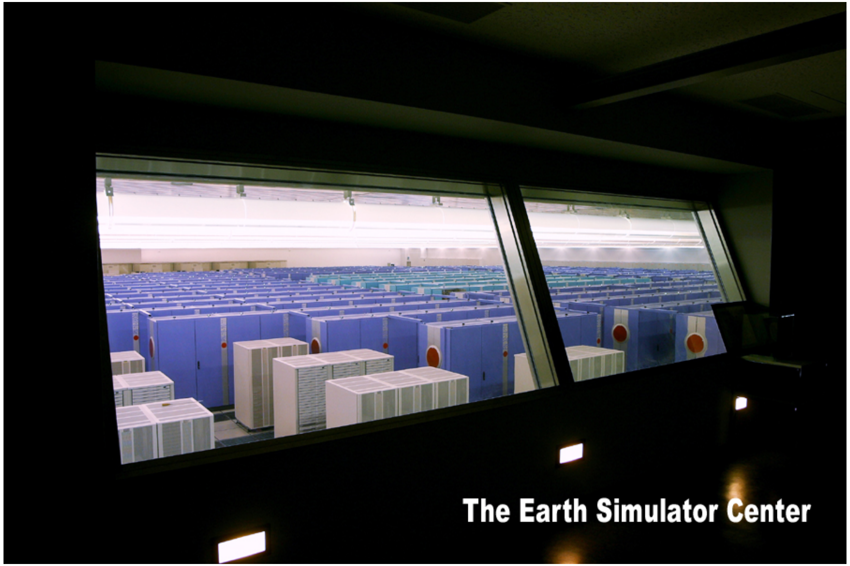
```
function F(n){
 Create an array fib[1..n]
 fib[1] = 1
 fib[2] = 1
 for i = 3 to n:
    fib[i] = fib[i-1] + fib[i-2]
 return fib[n]
}
```

The “right” question is: How does the running time grow?

E.g. How long does it take to compute F(200)?

….let’s say on….

NEC Earth Simulator



Can perform up to 40 trillion operations per second.

The running time of the recursive implementation

The Earth simulator needs 2^{92} seconds for F_{200}.

Time in seconds	Interpretation
2^{10}	17 minutes
2^{20}	12 days
2^{30}	32 years
2^{40}	cave paintings
2^{70}	The big bang!

```
function F(n){
    if(n == 1) return 1
    if(n == 2) return 1
   return F(n-1) + F(n-2)
}
```

Let's try calculating F_{200} using the iterative algorithm on my laptop…..

Goals for measuring time efficiency

- **Focus on the impact of the algorithm:**

Simplify the analysis of running time by ignoring “details” which may be an artifact of the underlying implementation:

  - E.g., 1000001 ≈ 1000000
  - Similarly, $3n^2 \approx n^2$

- **Focus on trends as input size increases (asymptotic behavior):**

How does the running time of an algorithm increases with the size of the input in the limit (for large input sizes)

Counting steps (instead of absolute time)

- Every computer can do some primitive operations in constant time:
  - Data movement (assignment) $x = 5;$
  - Control statements (branch, function call, return) $f();$
  - Arithmetic and logical operations $x > y$ $x + y$

- By inspecting the pseudo-code, we can count the number of primitive operations executed by an algorithm

as a function of input size N.

count of primitive operations.

Running Time Complexity

Step 1 : Count the primitive operations

Start by counting the primitive operations

```
/* N is the length of the array*/
int sumArray(int arr[], int N)
{
        int result=0;
        for(int i=0; i < N; i++)
                result+=arr[i]; (2)
        return result;
}
```

N times

Count

1

$\rightarrow 2N$

1

$2N+2$

$T(N) = 2N+2$

Big-O notation

$2N+2$

N	Steps = 5*N +3
1	8
10	53
1000	5003
100000	500003
10,000,000	50000003

$T(N) = \cancel{2}N + 200000$

$T(n) = an$ (Linear)

$T(n) = \cancel{a}n^2 + \cancel{b}$ (Quadratic)

- Simplification 1: Count steps instead of absolute time
- Simplification 2: Ignore lower order terms
  - Does the constant 3 matter as N gets large?
- Simplification 3: Ignore constant coefficients in the leading term (5*N) simplified to N

After the simplifications,

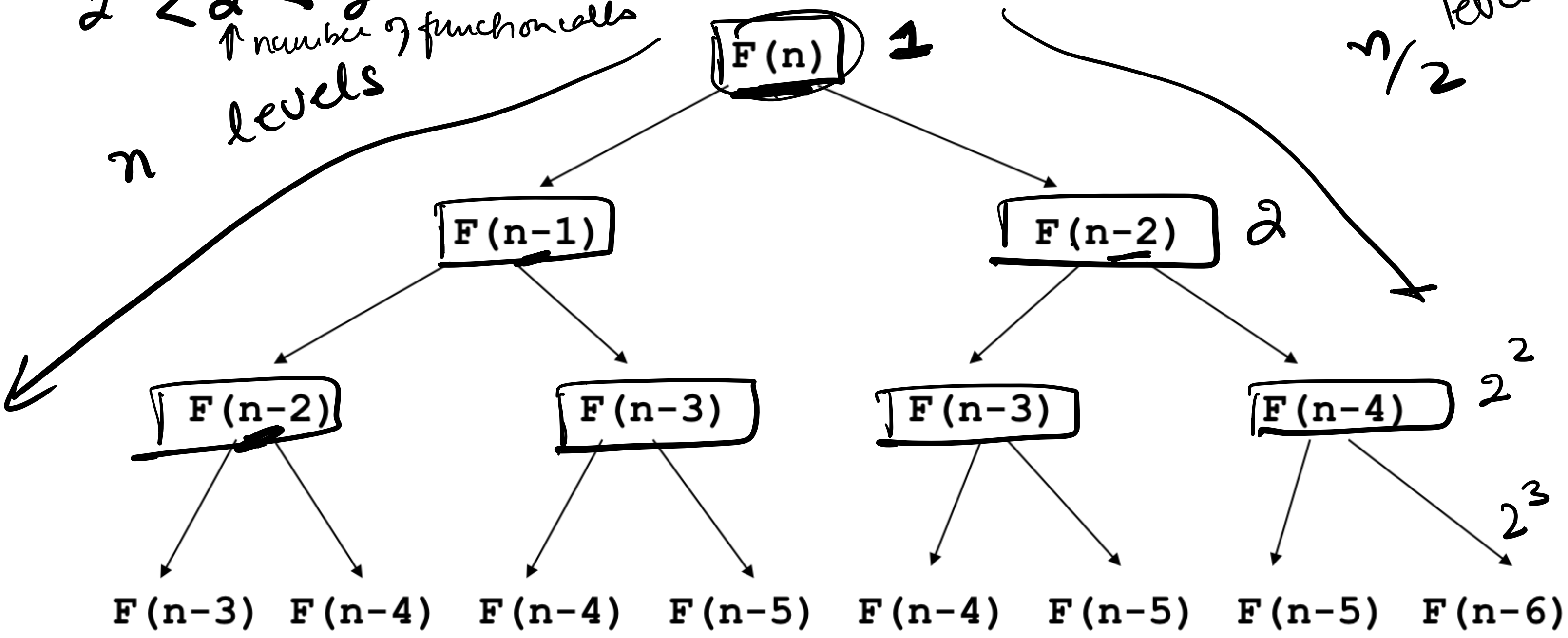
The number of steps grows linearly in N
Running Time = O(N) pronounced "Big-Oh of N"

Drop some terms that grow slowly compared to other terms

What takes so long? Let's unravel the recursion…

$2^{n/2} < 2^{0.69n} < 2^n$

↑ number of function calls



The same subproblems get solved over and over again!

Orders of growth

Big-Oh Analysis

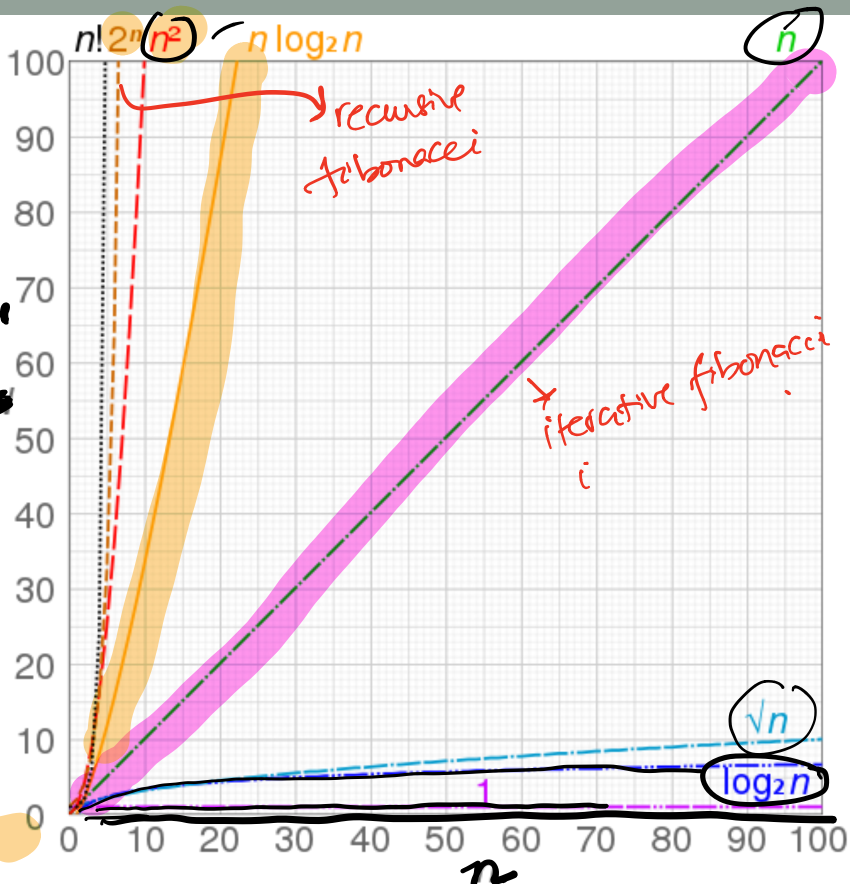
- We are interested in how algorithm running time scales with input size

Running time

- Big-Oh notation allows us to express that by ignoring the details

- 20n hours v. n^2 microseconds:
  - *which has a higher order of growth?*
  - *Which one is better?*

$T(n) = 2^n + 50{,}000\,n + 100 + 5n^2$



$O(2^n) \rightarrow$ function

Big-O notation lets us focus on the big picture

Recall our goals:

- **Focus on the impact of the algorithm**
- **Focus on asymptotic behavior (running time as N gets large)**

Count the number of steps in your algorithm: 3+ 5*N
Drop the constant additive term : 5*N
Drop the constant multiplicative term : N
Running time grows linearly with the input size
Express the count using **O-notation**
Time complexity = O(N)

Given the step counts for different algorithms, express the running time complexity using Big-O

1. 10,000,000 O(1) constant time
2. 3*N O(N)
3. 6*N-2 O(N)
4. 15*N + 44 O(N)
5. 50*N*logN O(N log N)
6. N^2 O(N^2)
7. N^2-6N+9 O(N^2)
8. $3N^2$+4*log(N)+1000 O(N^2)

For polynomials, use only leading term, ignore coefficients: linear, quadratic

Common sense rules of Big-O

1. Multiplicative constants can be omitted: $14n^2$ becomes n^2 .
2. n^a dominates n^b if $a > b$: for instance, n^2 dominates n.
3. Any exponential dominates any polynomial: 3^n dominates n^5 (it even dominates 2^n).

What is the Big O of sumArray2

A. $O(N^2)$
B. $O(N)$
C. $O(N/2)$
D. $O(\log N)$
E. None of the array

```
/* N is the length of the array*/
int sumArray2(int arr[], int N)
{
  1     int result=0;
  N/2   for(int i=0; i < N; i=i+2)
      3     result+=arr[i];
  1     return result;
}
```

$T(N) = \cancel{1} + 3\frac{N}{2} + \cancel{1}$

$= O(N)$ drop constants and coefficients

What is the Big O of sumArray2

A. $O(N^2)$
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C. $O(N/2)$
D. $O(\log N)$
E. None of the array

```
/* N is the length of the array*/
int sumArray2(int arr[], int N)
{
        int result=0;
        for(int i=1; i < N; i=i*2)
                result+=arr[i];
        return result;
}
```

This affects the no. of times the loop executes

Iteration #	i
1	1
2	2
3	4
4	8
5	16
⋮	⋮
k	2^{k-1}

$2^{1-1} = 1$

$2^{2-1} = 2$

$2^{3-1} = 4$

$$2^{k-1} \geq N$$

$$(k-1) \geq \log_2 N$$

$$k \geq \boxed{\log_2 N + 1}$$

$$O(\log_2 N)$$

Operations on sorted arrays

- Min :
- Max:
- Median:
- Successor:
- Predecessor:
- Search**:**
- Insert :
- Delete:

6	**13**	**14**	**25**	**33**	**43**	**51**	**53**	**64**	**72**	**84**	**93**	**95**	**96**	**97**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**

lo **hi**

Next time

- Running time analysis of Binary Search Trees

References:
https://cseweb.ucsd.edu/classes/wi10/cse91/resources/algorithms.ppt
http://algorithmics.lsi.upc.edu/docs/Dasgupta-Papadimitriou-Vazirani.pdf