FAST LOOKUP WITH HASHTABLES

Problem Solving with Computers-II

Hastable — Dictionary like data structure

std::unordered_set<T> – stores unique keys, no duplicates

```
std::unordered_set<string> countries = {"USA", "France", "India"};
s.insert("Germany");
s.erase("France");
auto found = find("USA");
```

std::unordered_map<Key, Value> – stores key-value pairs (like a dictionary)

```
unordered_map<string, string> capitals= {{"USA", "Washington"}, {"France", “Paris"}};

capitals["Germany"] = "Berlin"; // insert
capitals.erase("France");       // delete
auto found = find("USA");       // find
```

Hashtable for fast lookup

Blockchain | Likes | Caching | Passwords

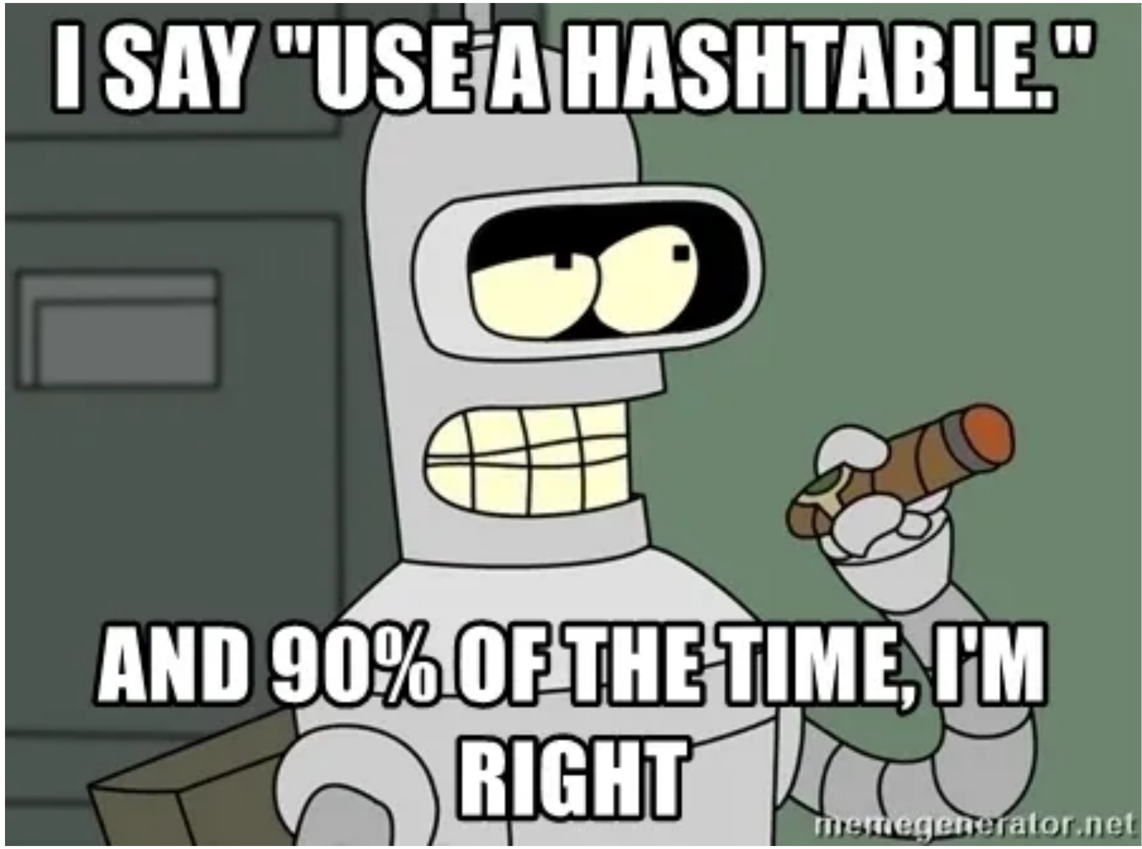
Dictionary-like data structure

Japan	: Tokyo
USA	: Washington
India	: New Delhi

All operations O(1) on average*:

- find
- insert
- erase

*no worst case guarantees but fast in practice!



Source: https://medium.com/@saurabh.bhoy910/stl-unordered-map-hashtable-c7054b28d07f

Amazon User Tracking Scenario

Amazon engineers track ~200M unique users by IP addresses during peak shopping season. Each website access requires checking if the IP is in the list; if not, it's added. This helps analyze user interest and personalize recommendations.

Which data structure should Amazon engineers use to track ~200M unique users by IP addresses?

Goal - Fast search (lookup) and insert

Which data structure should Amazon engineers use to track ~200M unique users by IP addresses?

- Keys: IP addresses (32 bits)
  - example : 192.168.1.6
- 2^32 ~ 4.3B possible IPs
- 200M (4.7%) unique users

192 (8 bits)	168 (8 bits)	1 (8 bits)	6 (8 bits)

`192.168.1.6 → uint32_t: 3234251782`

Can we achieve O(1) search?

Naïve approach: 2^{32}-sized vector

If you had to track unique IPs out of 4.3 billion (2^32) possibilities, how much memory do you think you'd need if you used a direct index approach?

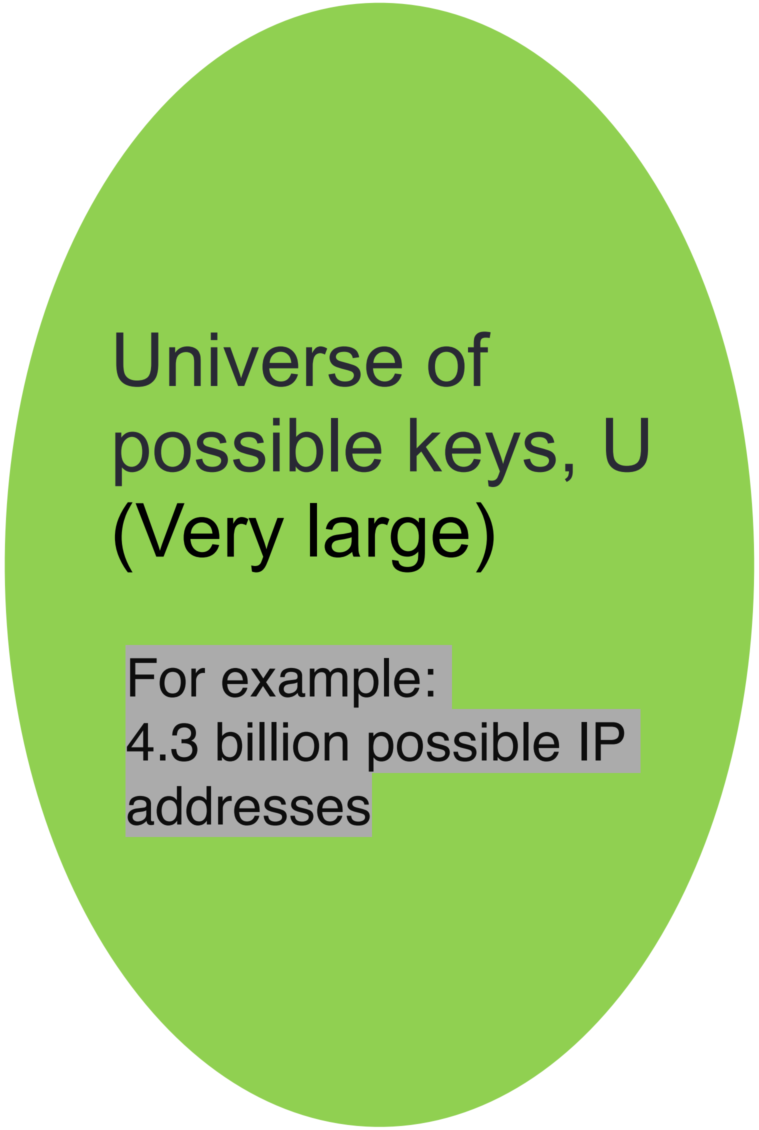
2^10 bytes = 1KB, 2^20 bytes = 1 MB, 2^30 bytes = 1GB

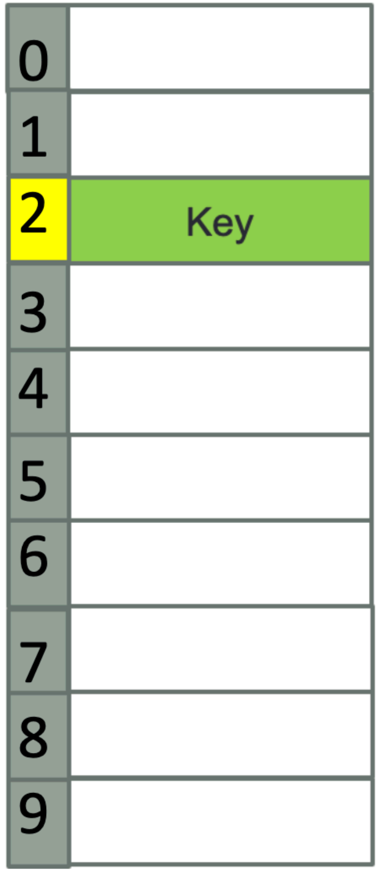
Assuming 1 bit per IP, we need 2^32 bits = 2^ 29 bytes = 512 MB

Assuming 4 bytes per IP = 512 * 32 MB = 16 GB

Setup for hash tables

- Keep track of evolving set S whose size is much less than the universe of all possible keys
- For example, 200M unique users (~ 5% of all possible IPs)

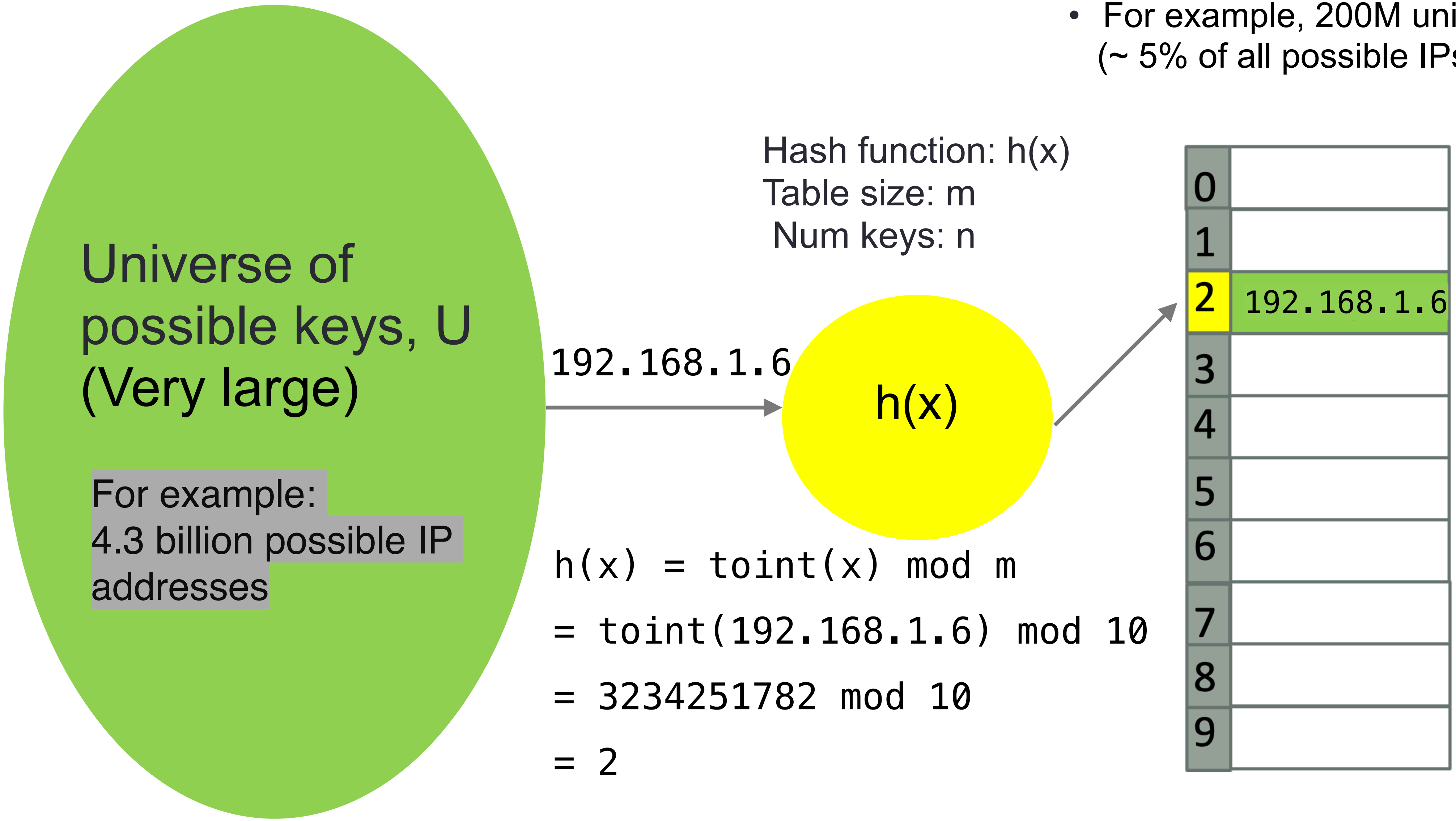




Insert key 192.168.1.6

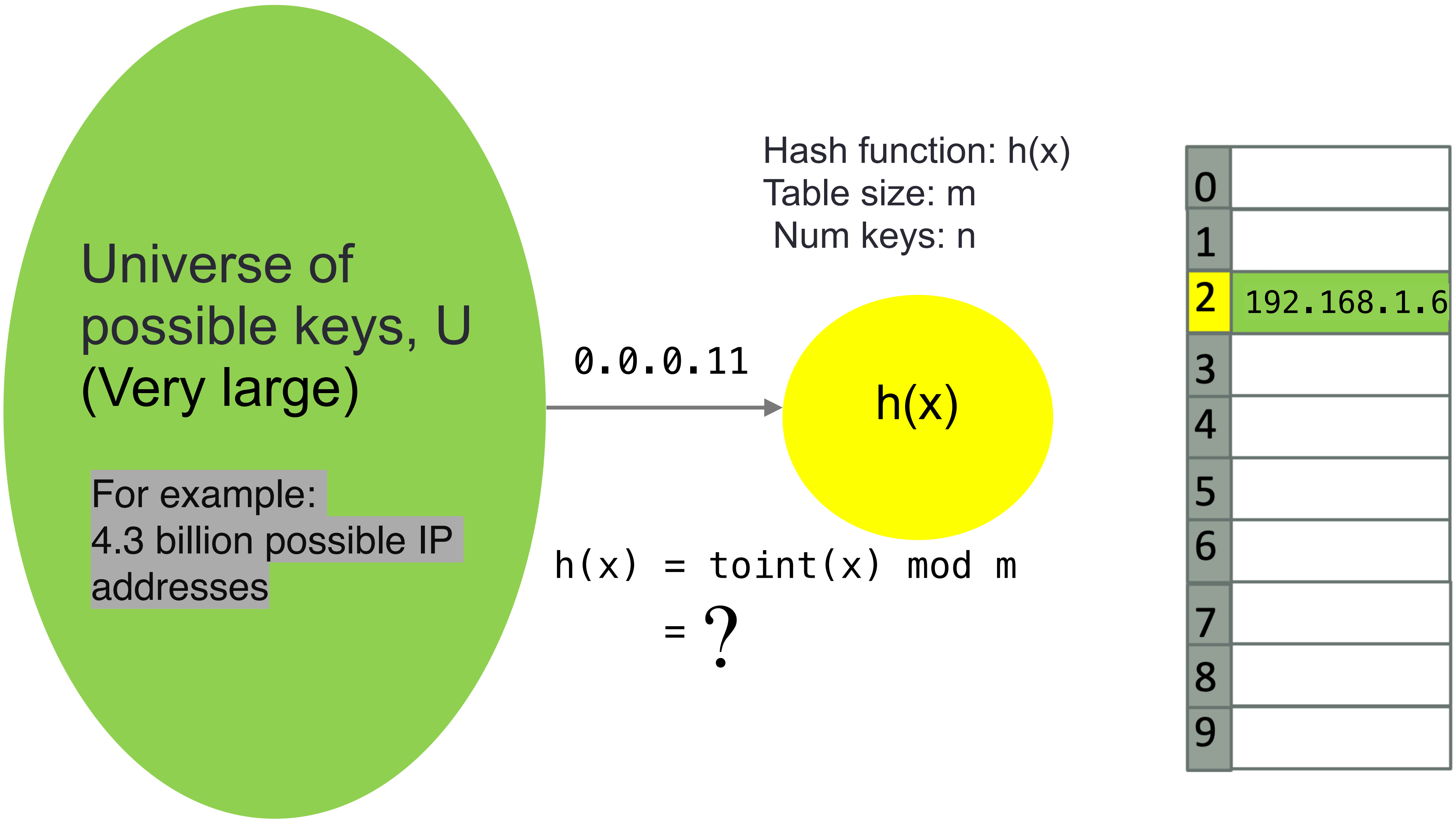
- Keep track of evolving set S whose size is much less than the universe of all possible keys
- For example, 200M unique users (~ 5% of all possible IPs)

Hash function: h(x)
Table size: m
Num keys: n

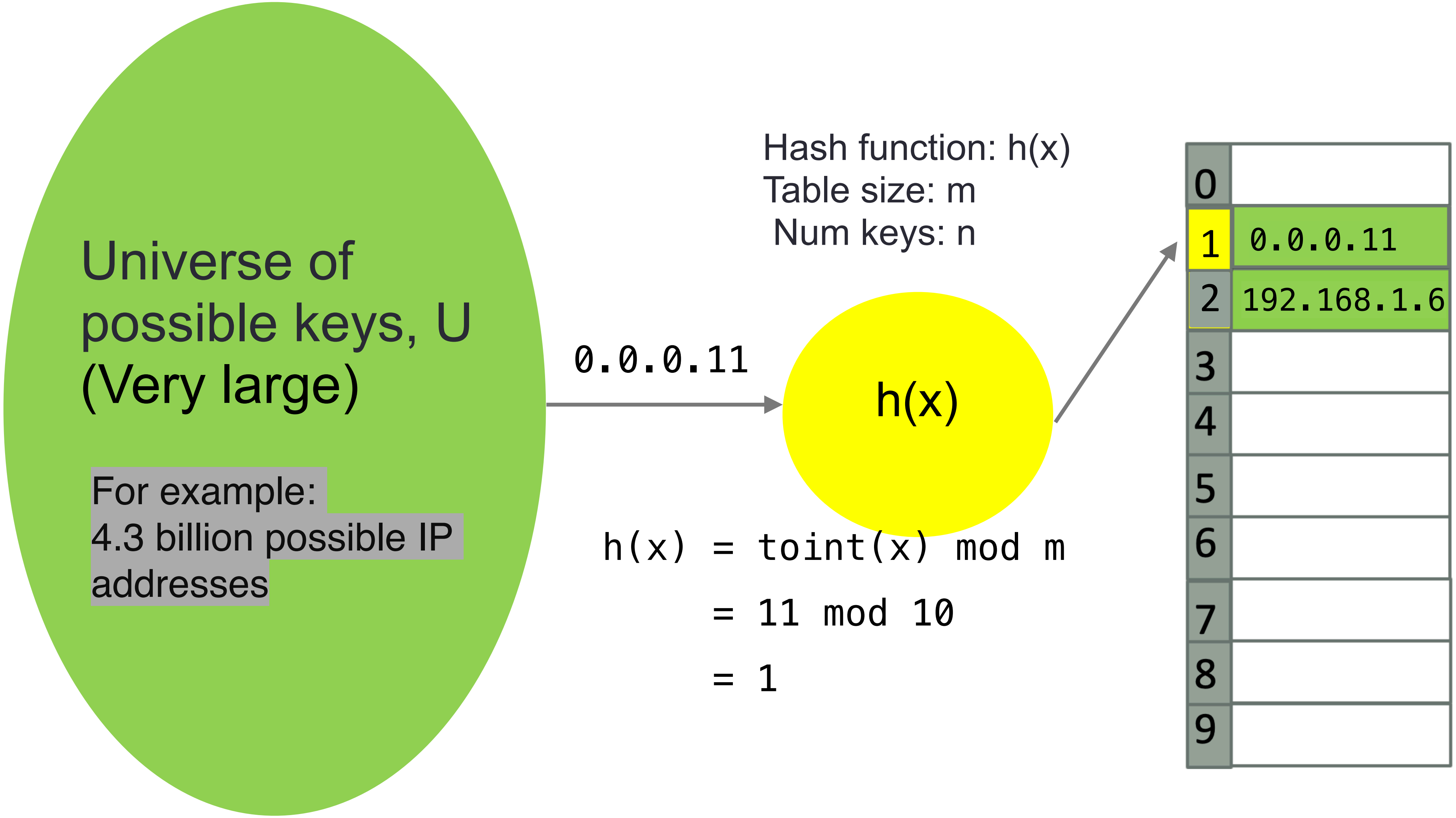



```
h(x) = toint(x) mod m
= toint(192.168.1.6) mod 10
= 3234251782 mod 10
= 2
```

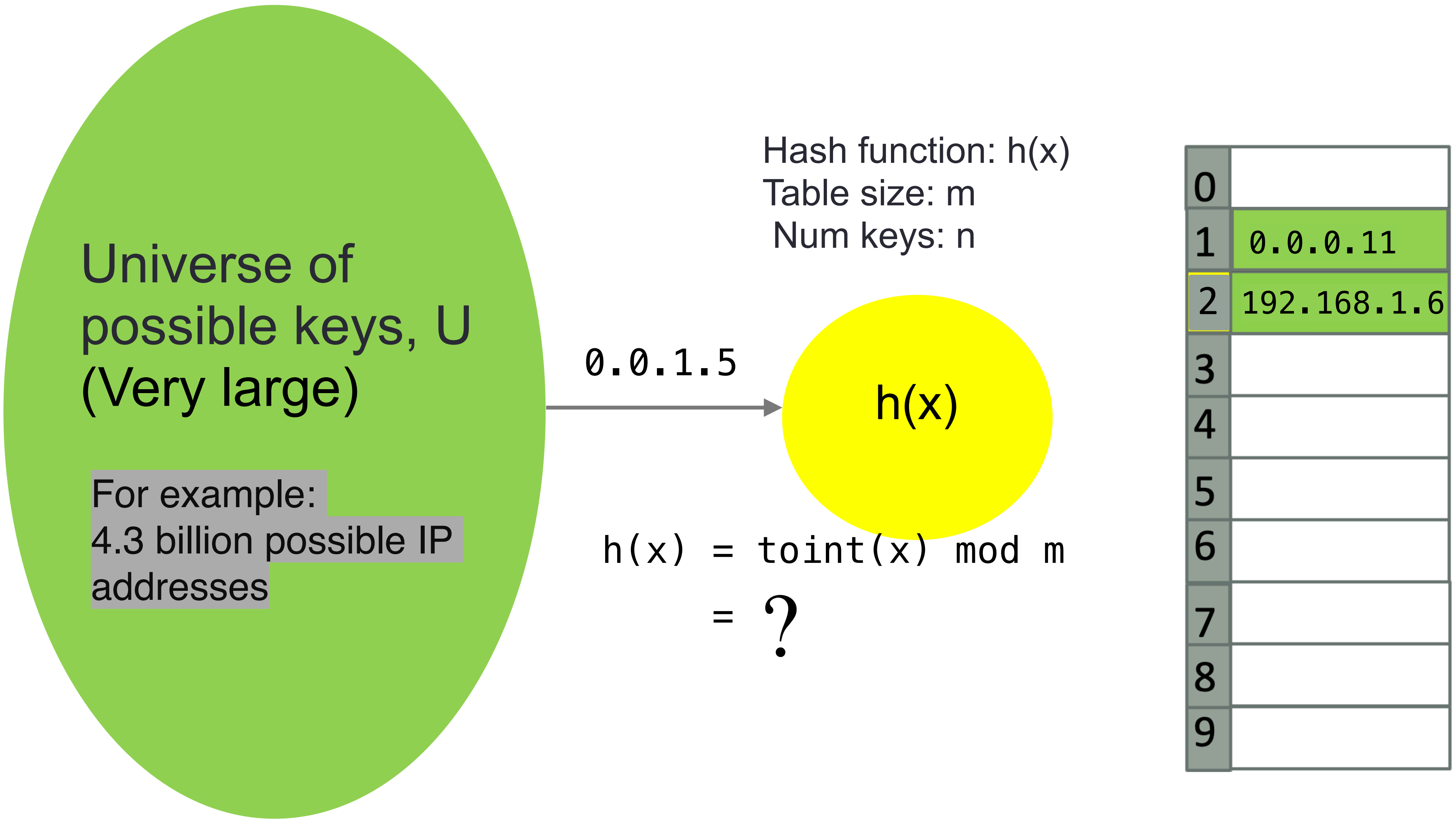
Insert key 0.0.0.11



Insert key 0.0.0.11



Insert 0.0.1.5


Universe of possible keys, U (Very large)
For example: 4.3 billion possible IP addresses
0.0.1.5
Hash function: h(x)
Table size: m
Num keys: n
h(x)
h(x) = toint(x) mod m
= ?
0
1 0.0.0.11
2 192.168.1.6
3
4
5
6
7
8
9

Insert key 0.0.1.5 results in a collision!

What should happen when two IPs hash to the same index?

A	*B*	*C*	*D*
Crash	Overwrite	Chain	Reject

Universe of possible keys, U (Very large)

For example:
4.3 billion possible IP addresses

Hash function: h(x)
Table size: m
Num keys: n

`0.0.1.5` → h(x)

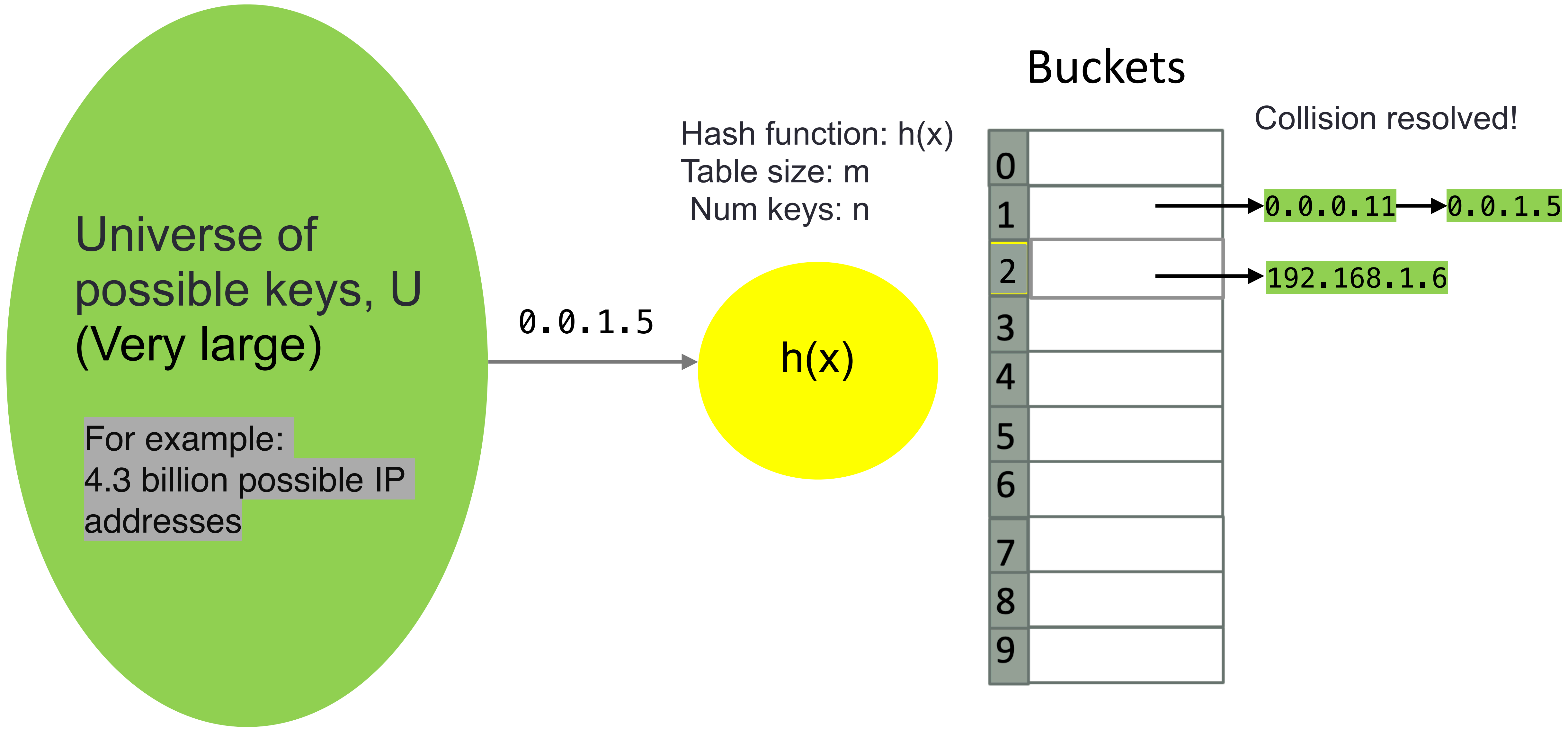
```
h(x) = toint(x) mod m
     = (256 + 5) mod 10
     = 261 mod 10
     = 1
```

0	
1	`0.0.0.11`
2	`192.168.1.6`
3	
4	
5	
6	
7	
8	
9	

`0.0.1.5`

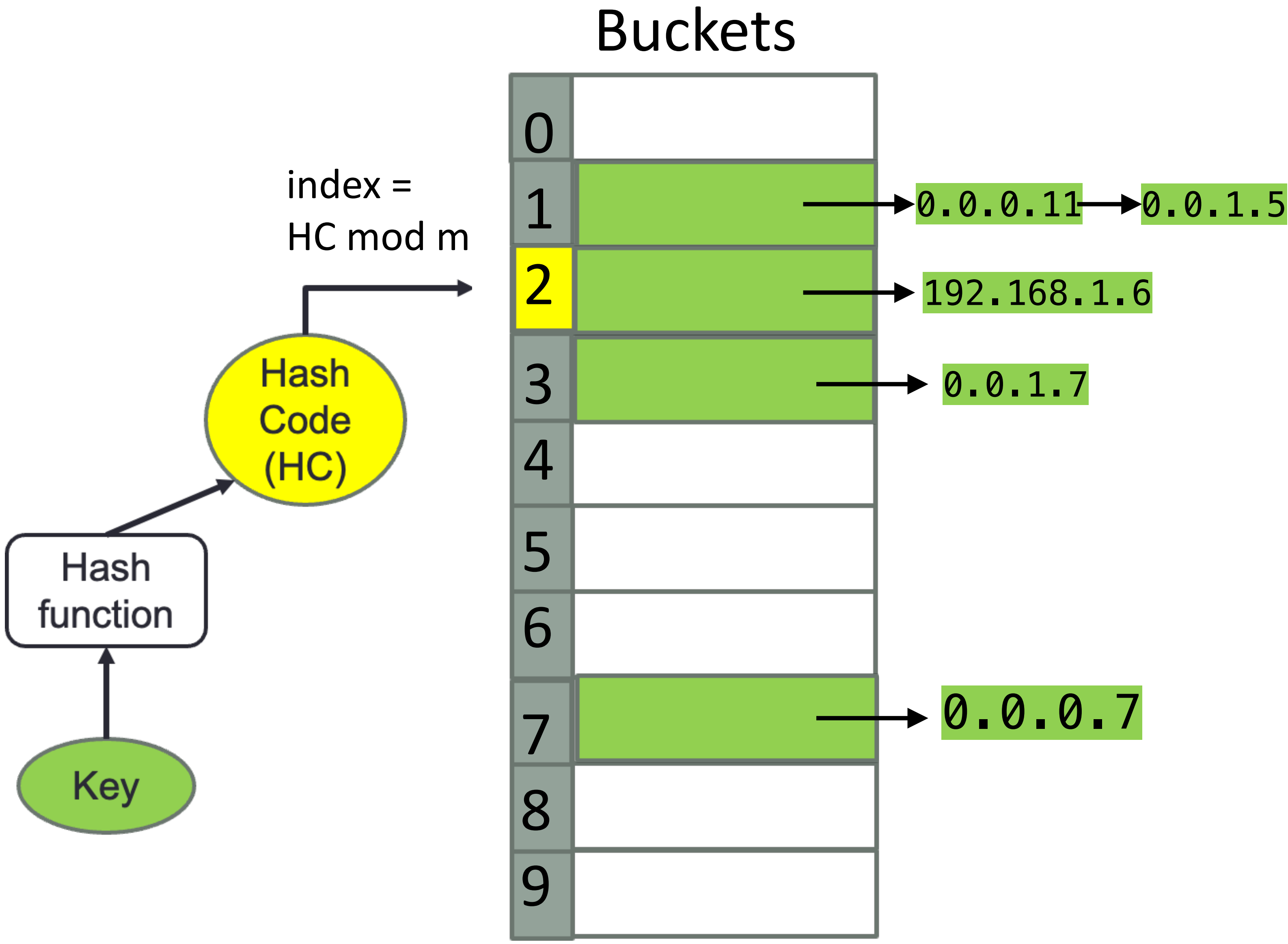
Collision: Two keys map to the same spot!

Resolving collisions: separate chaining



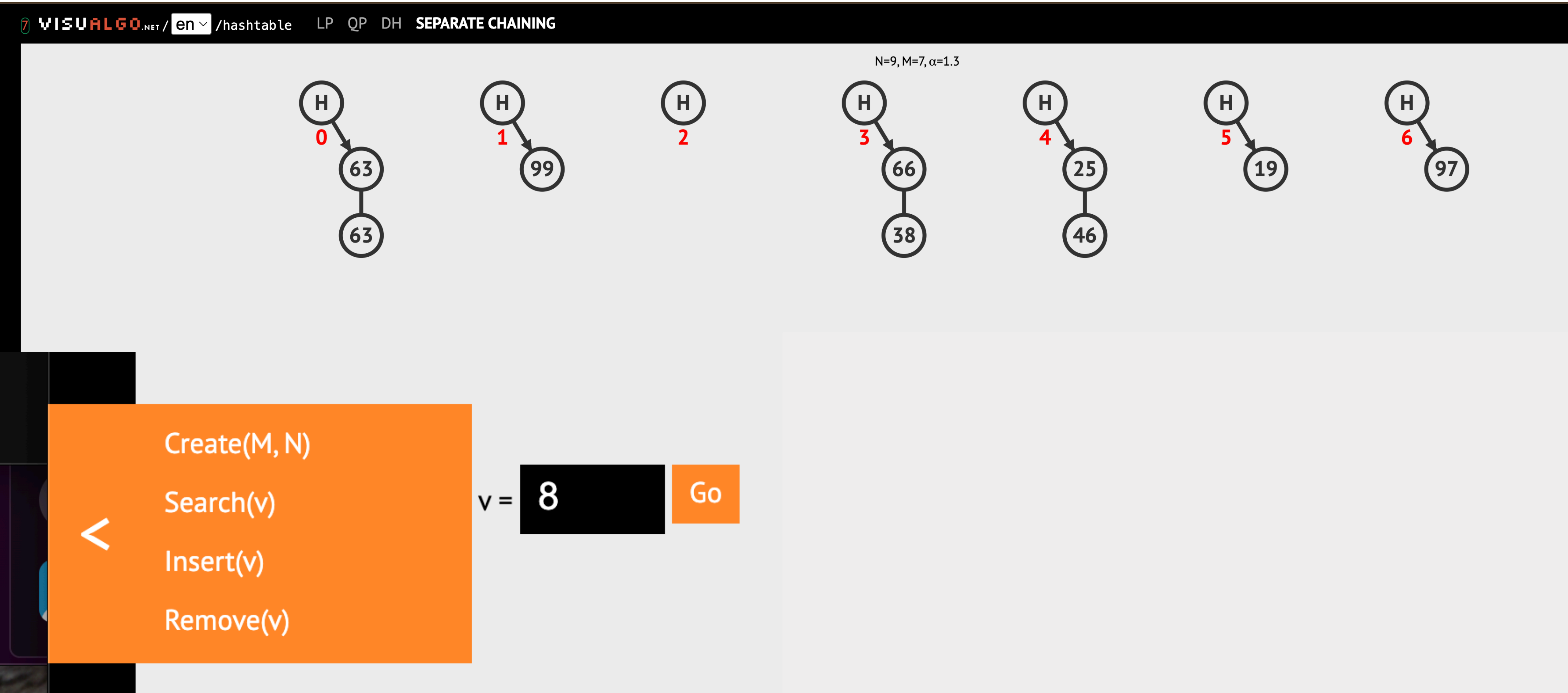
(Refined) Logical model of a hash table

- Keys stored in buckets (vector)
- Keys used to compute index of position in vector
- Each bucket can store multiple keys as a linked list
- Hashtable with separate chaining: Vector of linked list



Hashtable visualization

https://visualgo.net/en/hashtable



Goal - Fast search (lookup) and insert

Which data structure should Amazon engineers use to track ~200M unique users by IP addresses?

A) Set (Balanced BST)
B) unordered_set (Hashtable)
C) Priority Queue
D) Queue
E) Vector with 2^32 entries (one for each possible IP address)

192 (8 bits)	168 (8 bits)	1 (8 bits)	6 (8 bits)

```
192.168.1.6 → uint32_t: 3234251782

Total IPs: 2^32 ≈ 4.3B
```

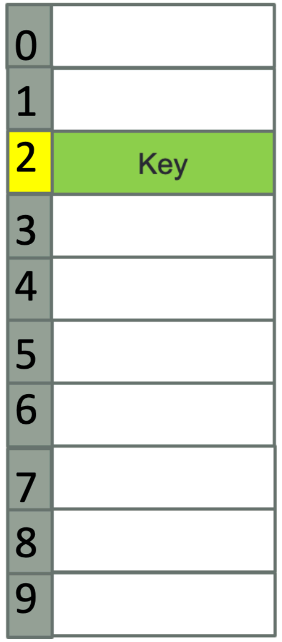
Design challenges

- **Deciding on collision resolution strategy**
- **Deciding the size of hash table**
- **Deciding the hash function**

Universe of possible keys, U (Very large)

For example:
4.3 billion possible IP addresses

Keep track of evolving set S whose size is much less than the universe of all possible keys



For example, 200M unique users
(~ 5% of all possible IPs)

Can we guarantee good performance?

We implemented a hashtable with separate chaining.

Table size: m
Number of keys: n
Load factor α = n/m
Hash function: h(x) = x mod m

If randomly uniformly selected keys, then “on average” things seem fine.
Expected time to search is $O(\alpha)$ (in practice, choose $\alpha = 1$)

Can all keys still hash to the same bucket?
(Worst-case scenario, linear search complexity!)

What's the chance of linear search complexity?

What is the probability that *all* keys hash to the *same* bucket?
Let's say we're inserting **n = 1000** keys into an empty table with **m = 1000** buckets.

A. 0

B. $1/1000$

C. $1/1000^{999}$

D. $1/1000^{1000}$

E. It depends…

- We're using the hash function **h(x) = x mod m**
- Keys are chosen independently and uniformly at random
- Hash table uses separate chaining

Is there a hash function that avoids linear search complexity?

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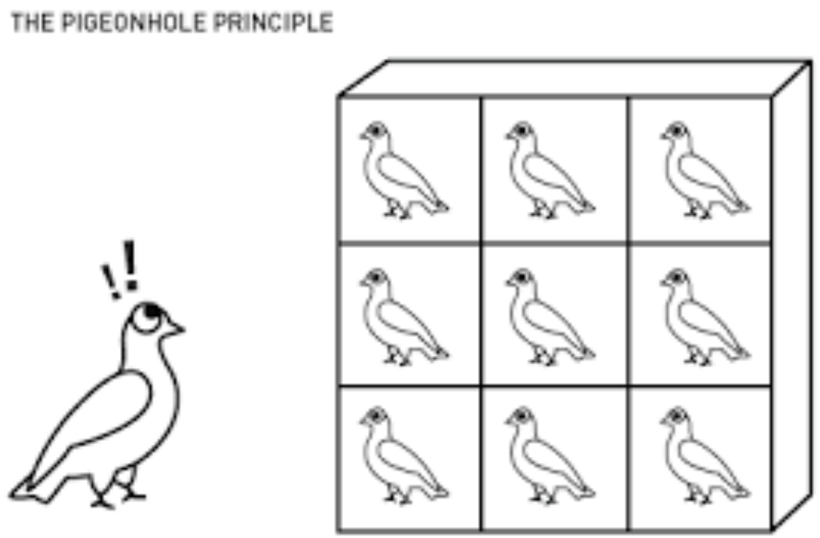
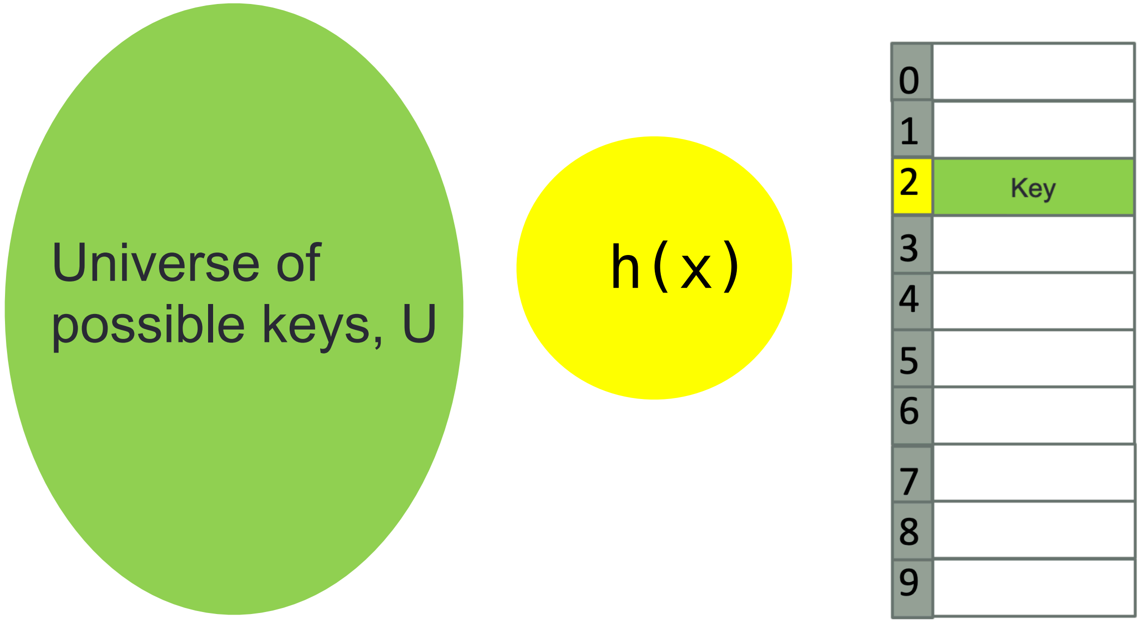


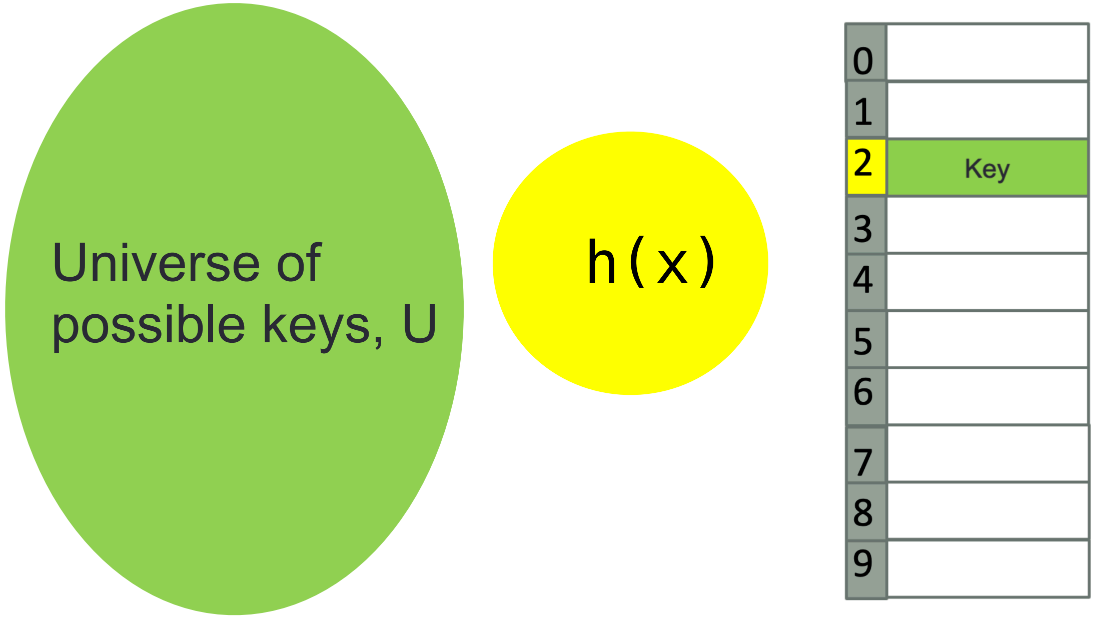


Pigeonhole Principle:
If there are m pigeonholes and m + 1 pigeons, at least one pigeonhole has more than 1 pigeon

Generalized Pigeonhole Principle:
If there are m pigeonholes and nm + 1 pigeons, at least one pigeonhole has more than__________ pigeons.

Is there a hash function that avoids linear search complexity?

Theorem: Suppose a hash table of size m is used to store a set S of n keys drawn from the universe U, where $|U| > nm$. Then, *no matter which hash function* $h : U \rightarrow \{0,1,2,\ldots m-1\}$ is chosen, there is a set $S \subset U$ of n keys that all map tp the same location.



Proof:

1. Pick any hash function of your choosing.
2. Map all keys of U using h to the table of size m.
3. By the pigeonhole principle, at least one table slot gets at least n keys because $|U| > nm$
4. Choose those n keys as input set S
5. Now h will map S to a single location.

Generalized Pigeonhole Principle:
If there are m pigeonholes and nm + 1 pigeons, at least one pigeonhole has more than n pigeons.

What does this mean for real-world applications?

Universal Hash Functions

Main result so far: No single hash function can avoid worst case linear time search complexity!

Main idea behind universal hash functions: Don't fix a single hash function. Choose one randomly from a "good" family of hash functions.

- Imagine an adversary picks **two keys**, x and y.
- You, the algorithm designer, get to **randomly choose** a hash function h from a big family H.
- The hash function is designed so that:
  The chance that $h(x) = h(y)$ is **at most 1/m**.

Example: $h(x) = ax + b \pmod{p}$ where p is a prime number

References

Professor Subhash Suri's CS 130A handout on hash tables:

https://sites.cs.ucsb.edu/~suri/cs130a/Hashing.pdf