BINARY SEARCH TREES

Problem Solving with Computers-II

Binary Search Trees

- What are the operations supported?
- What are the running times of these operations?

Complexity
Next Lectures :

- How do you implement the BST i.e. operations supported by it?

All the operations supported by a sorted Array

+ faster insert & delete

Operations supported by sorted arrays and Binary Search Trees

Search !

min

max

delete

insert

count

predecessor → next maller value

successor → next auger value

print in order

Operations supported by Sorted arrays and Binary Search Trees (BST)

Operations	Sorted Array	BST
Min	✓	✓
Max	✓	✓
Successor (next larger value)	✓	✓
Predecessor (next smaller value)	✓	✓
Search	✓	✓
Insert	✓	✓
Delete	✓	✓
Print elements in order	✓	✓

Binary Search

- Binary search. Given `value` and sorted array `a[]`, find index `i` such that `a[i] = value`, or report that no such index exists.
- Invariant. Algorithm maintains `a[lo]` ≤ `value` ≤ `a[hi]`.

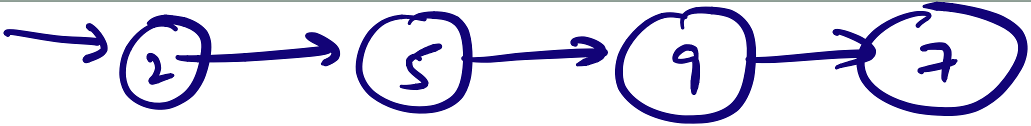
found it

- Ex. Binary search for 33.

6	13	14	25	33	43	51	53	64	72	84	93	95	96	97
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
lo			mid		hi	hi								hi
			lo											

$$\text{mid} = \left(\frac{lo+hi}{2}\right)$$

head

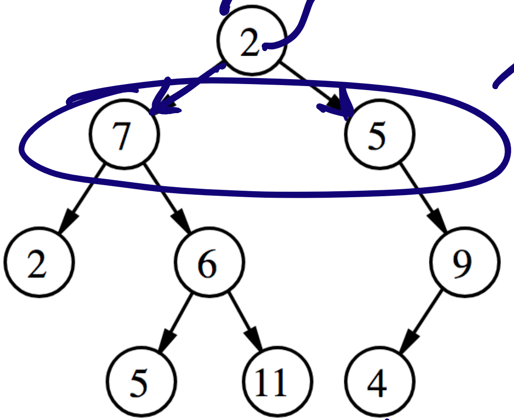


Trees

root

key value

7,5 are the children of 2



A tree has following general properties:

- One node is distinguished as a **root**;
- Every node (exclude a root) is connected by a directed edge *from* exactly one other node;

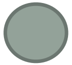
  A direction is: *parent -> children*
- *Leaf node: Node that has no children*

node with no children :

Leaf:

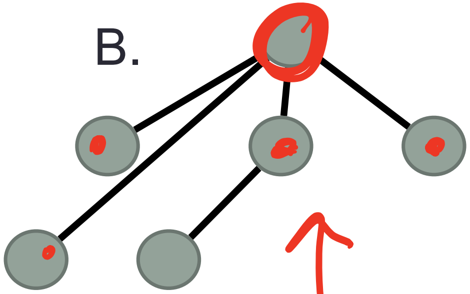
Which of the following is/are a tree?

A.



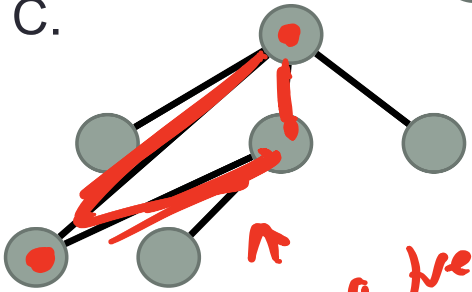
Tree with one node

B.



Valid

C.

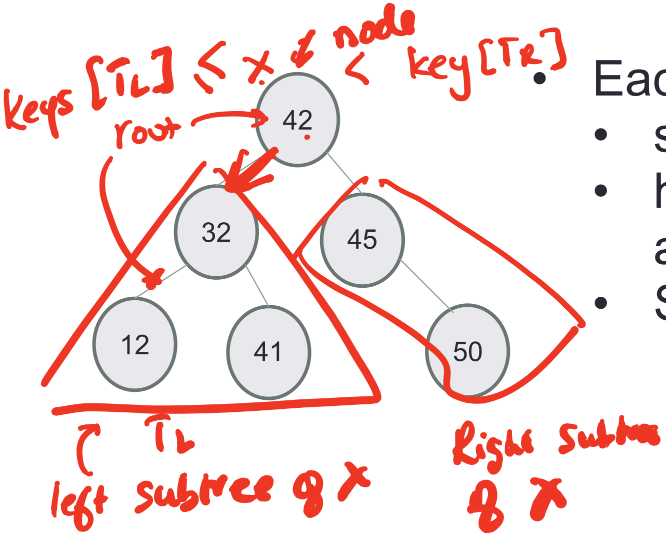


no loops in a tree

D. A & B

E. All of A-C

Binary Search Tree – What is it?



Each node:

- stores a key (k)
- has a pointer to left child, right child and parent (optional)
- Satisfies the Search Tree Property

For any node,
Keys in node's left subtree <= Node's key
Node's key < Keys in node's right subtree

Do the keys have to be integers?

```
struct Node {
    int data;
    Node * next;

}
```

Node in linked list

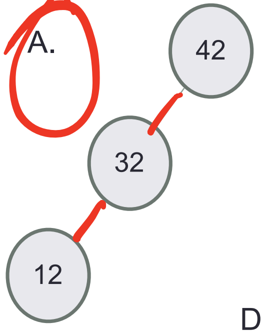
```
struct BSTNode {
    int data;
    BSTNode* left;
    BSTNode * right;
    BSTNode * parent;
};
```

BSTNode * root;

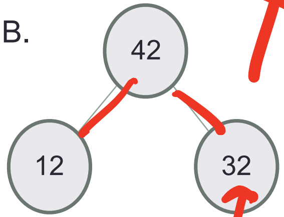
member variable of class BST

Which of the following is/are a binary search tree?

A.

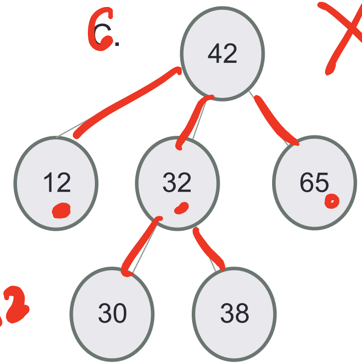


B.

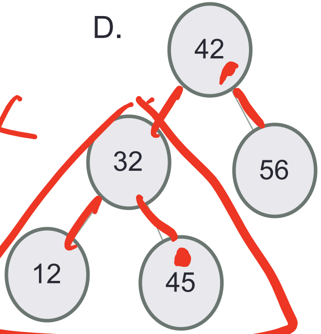


32 to the right of 42

C.

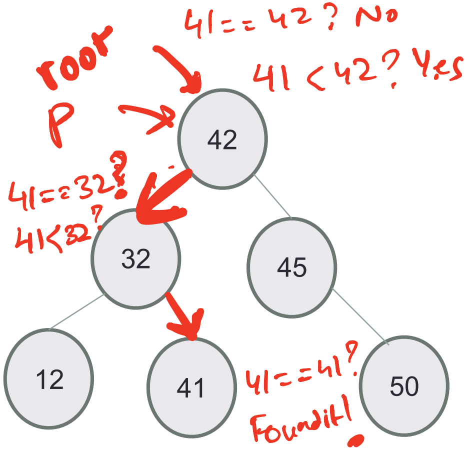


D.



E. More than one of these

BSTs allow efficient search!



- Start at the root;
- Trace down a path by comparing **k** with the key of the current node x:
  - If the keys are equal: we have found the key
  - If **k** < key[x] search in the left subtree of x
  - If **k** > key[x] search in the right subtree of x

```
if (p->data == value)
    return true;
if (p->data > value)
    return search(p->left, value);
return search(p->right, value);
```

Search for 41, then search for 53

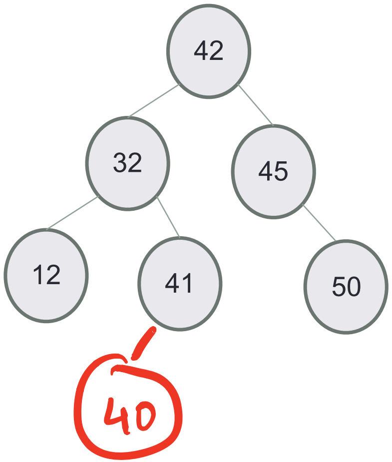
A node in a BST

```
class BSTNode {

public:
   BSTNode* left;
   BSTNode* right;
   BSTNode* parent;
   int const data;

   BSTNode( const int & d ) : data(d) {
      left = right = parent = 0;
   }
};
```

Define the BST ADT

Operations
Min
Max
Successor
Predecessor
Search
Insert
Delete
Print elements in order

Insert



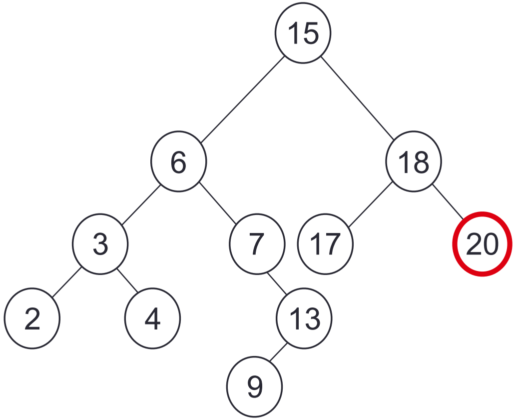
- Insert 40
- Search for the key (40)
- Insert at the spot you expected to find it

Max

Goal: find the maximum key value in a BST

Following right child pointers from the root, until a leaf node is encountered. The least node has the max value

Alg: `int BST::max()`



Maximum = 20

Min

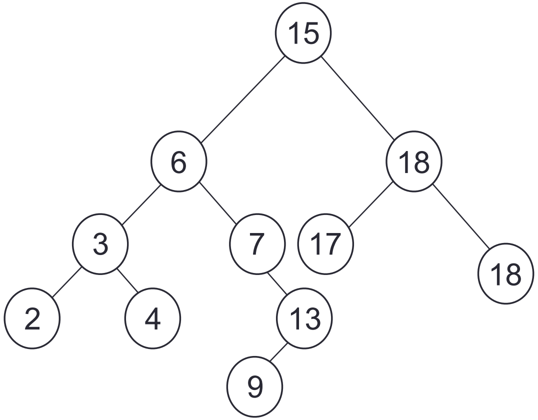
Goal: find the minimum key value in a BST

Start at the root.

Follow left________ child pointers from the root, until a leaf node is encountered

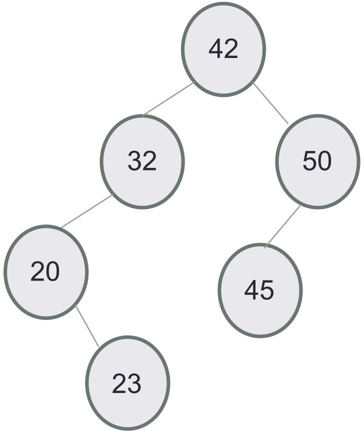
Leaf node has the min key value

Alg: `int BST::min()`



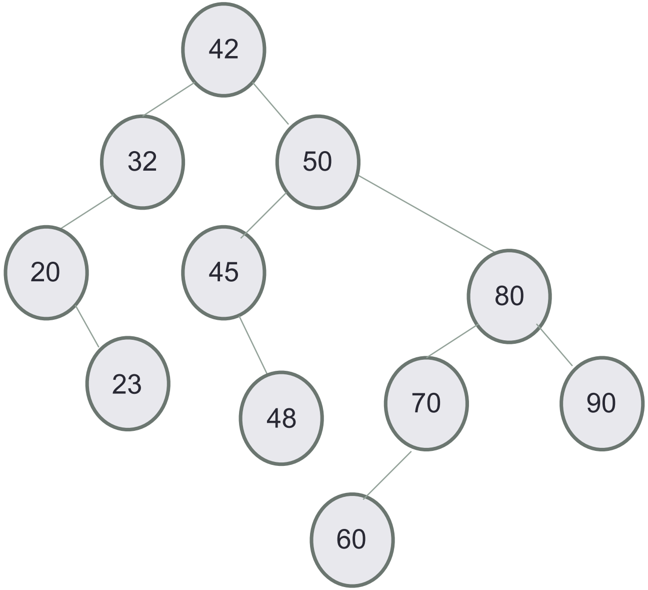
Min = ?

Predecessor: Next smallest element



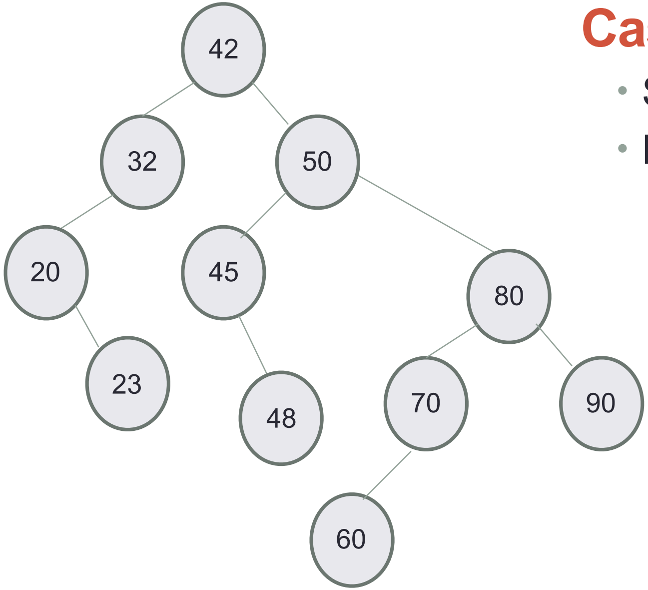
- What is the predecessor of 32?
- What is the predecessor of 45?

Successor: Next largest element



- What is the successor of 45?
- What is the successor of 50?
- What is the successor of 60?

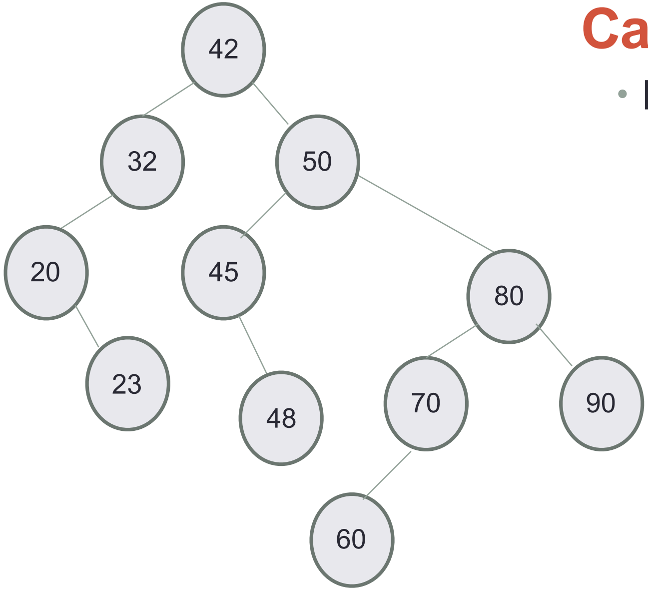
Delete: Case 1



Case 1: Node is a leaf node

- Set parent’s (left/right) child pointer to null
- Delete the node

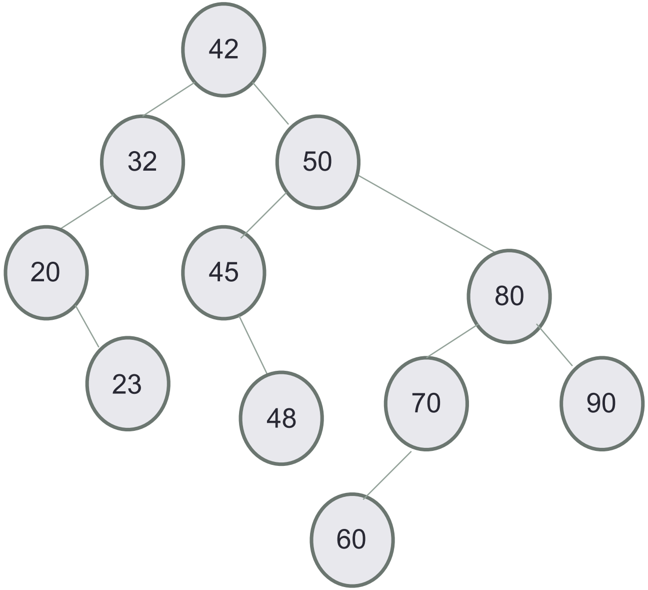
Delete: Case 2



Case 2 Node has only one child

- Replace the node by its only child

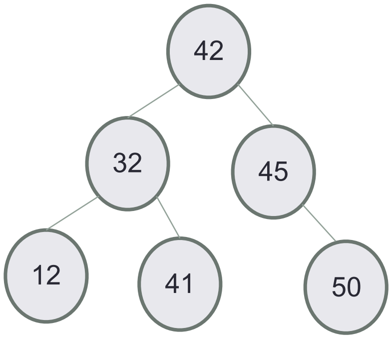
Delete: Case 3



Case 3 Node has two children

- Can we still replace the node by one of its children? Why or Why not?

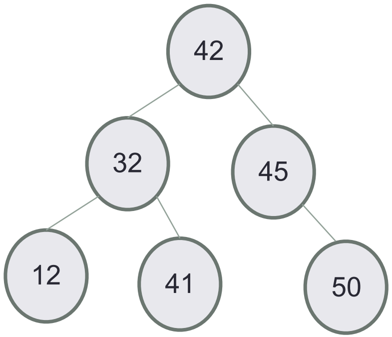
In order traversal: print elements in sorted order



Algorithm Inorder(tree)

1. Traverse the left subtree, i.e., call Inorder(left-subtree)
2. Visit the root.
3. Traverse the right subtree, i.e., call Inorder(right-subtree)

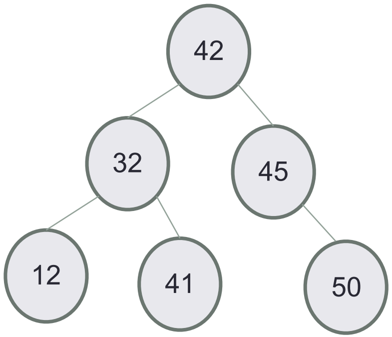
Pre-order traversal: nice way to linearize your tree!



Algorithm Preorder(tree)

1. Visit the root.
2. Traverse the left subtree, i.e., call Preorder(left-subtree)
3. Traverse the right subtree, i.e., call Preorder(right-subtree)

Post-order traversal: use in recursive destructors!



Algorithm Postorder(tree)

1. Traverse the left subtree, i.e., call Postorder(left-subtree)
2. Traverse the right subtree, i.e., call Postorder(right-subtree)
3. Visit the root.

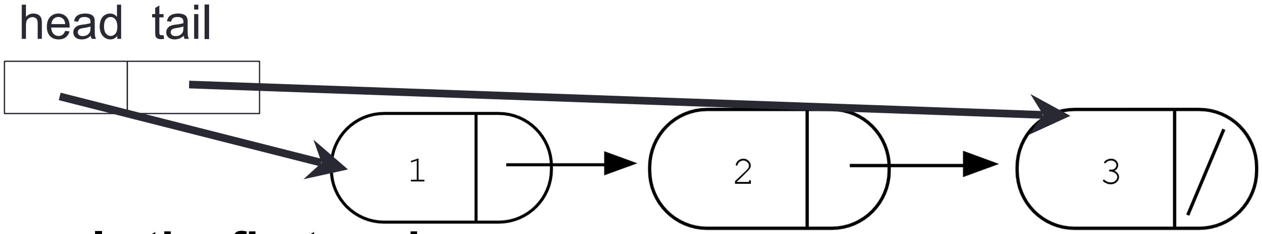
Concept Question

```
LinkedList::~LinkedList(){
   delete head;
}
```

```
class Node {
    public:
       int info;
       Node *next;
};
```

Which of the following objects are deleted when the destructor of Linked-list is called?

(A)



(B): only the first node

(C): A and B

(D): All the nodes of the linked list

(E): A and D

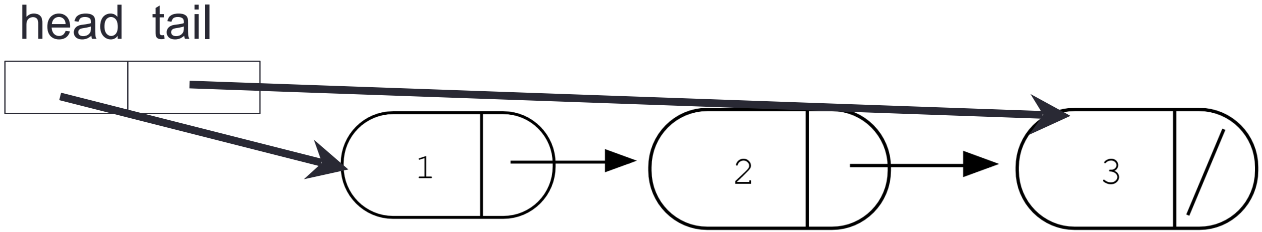
Concept Question

```
LinkedList::~LinkedList(){
   delete head;
}
```

```
Node::~Node(){
   delete next;
}
```

Which of the following objects are deleted when the destructor of Linked-list is called?

(A)



(B): All the nodes in the linked-list

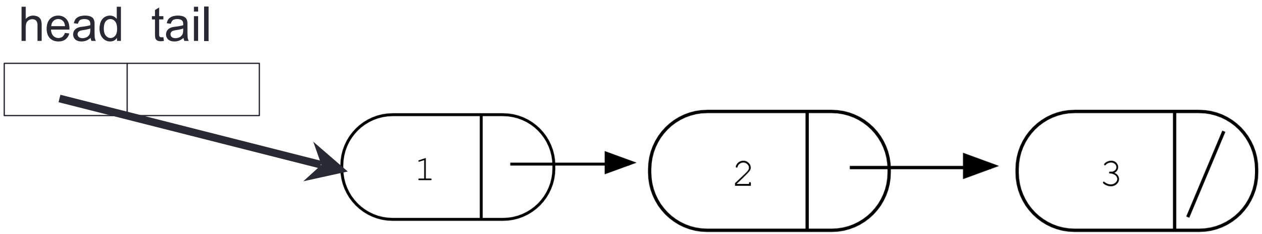
(C): A and B

(D): Program crashes with a segmentation fault

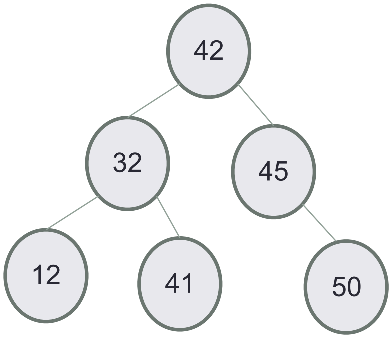
(E): None of the above

```
LinkedList::~LinkedList(){
   delete head;
}
```

```
Node::~Node(){
   delete next;
}
```



Post-order traversal: use in recursive destructors!



Algorithm Postorder(tree)

1. Traverse the left subtree, i.e., call Postorder(left-subtree)
2. Traverse the right subtree, i.e., call Postorder(right-subtree)
3. Visit the root.