

BINARY SEARCH TREES (CONTD)

Problem Solving with Computers-II

C++

```
#include <iostream>
using namespace std;

int main(){
    cout<<"Hola Facebook!";
    return 0;
}
```

A node in a BST

```
class BSTNode {
```

```
public:
```

```
    BSTNode* left;
    BSTNode* right;
    BSTNode* parent;
    int const data;
```

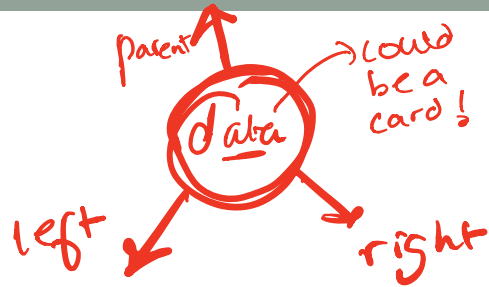
I type of data

```
    BSTNode( const int & d ) : data(d) {
        left = right = parent = 0;
    }
```

```
};
```

The data stored in any BST node could be of any type -
e.g. Card. as long as the operations $==$, $<$ and $>$
are defined on that type

```
struct Card {
    char suit;
    int value;
};
```



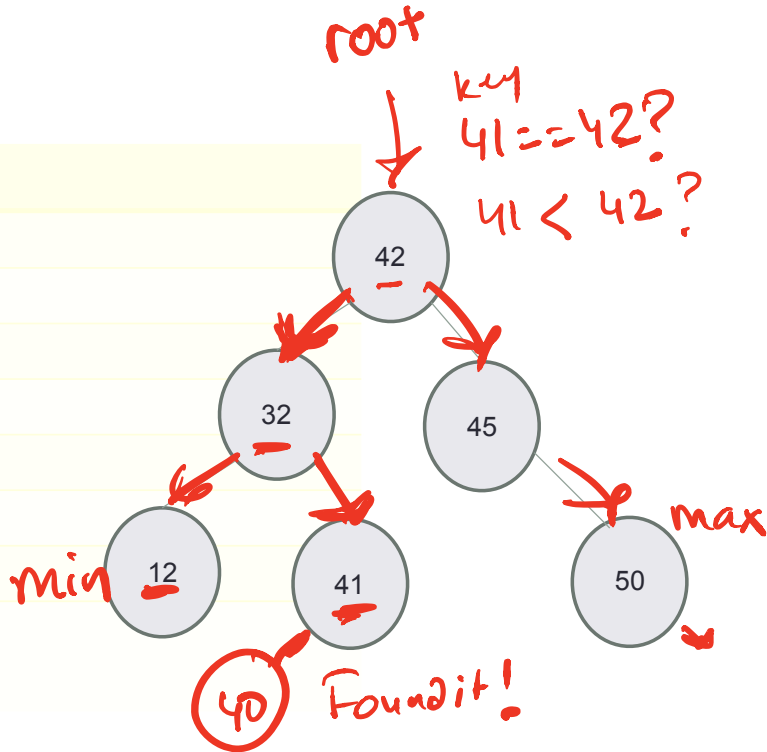
```
class BST {
```

```
private:
    BSTNode* root;
```

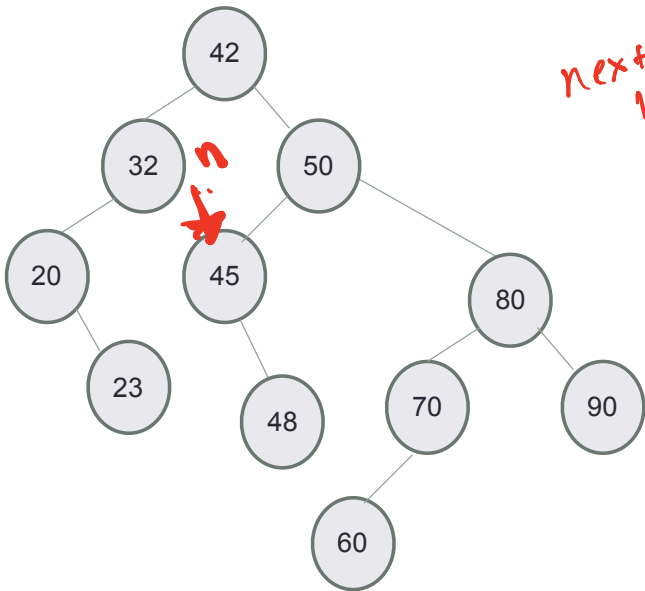
```
};
```

Define the BST ADT

Operations
Search ✓
Insert ✓
Min ✓
Max ✓
Successor ←
Predecessor
Delete
Print elements in order



Successor: Next largest element



next largest
key

↓
int

- What is the successor of 45?
- What is the successor of 50?
- What is the successor of 60?

key

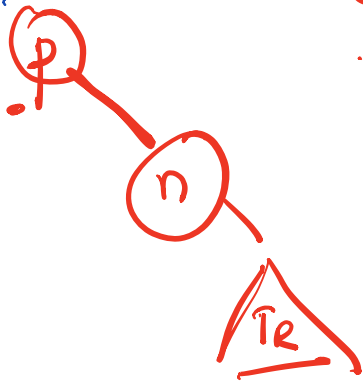
int successor (int value);

Our algorithm will be for a private version of this function that takes a pointer to a node as input and returns a pointer to the node with the next largest value.

Node* successor - Private (Node* n);

Case 1: n has a right subtree T_R

Case 1a



$$\text{key}(n) < \text{key}(T_R) \dots \textcircled{1}$$

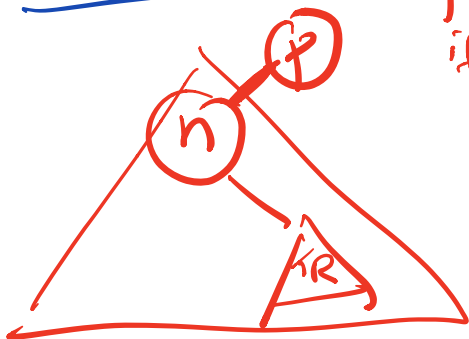
if n is parent's right child:

$$\text{key}(p) < \text{key}(n) \dots \textcircled{2}$$

From $\textcircled{1}$ & $\textcircled{2}$

$$\text{key}(p) < \text{key}(n) < \text{key}(T_R)$$

Case 1b



if n is its parent's left child
if $(n \rightarrow \text{parent} \rightarrow \text{left} == n)$:

$$\text{key}(n) < \text{key}(p) \dots \textcircled{3}$$

$$\text{key}(n) < \text{key}(T_R) \dots \textcircled{4}$$

How does $\text{key}(p)$ compare with $\text{key}(T_R)$

- A.
- B.
- C.

$$\text{key}(p) < \text{key}(T_R)$$

$$\text{key}(p) > \text{key}(T_R)$$

Can't say

- $\textcircled{5}$

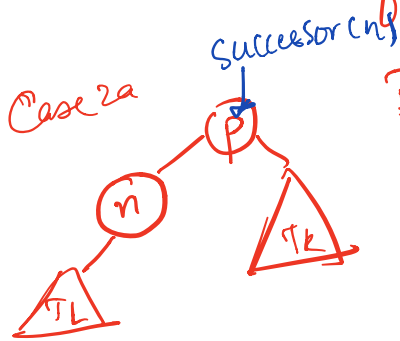
from (3) (4) (5)

$$\text{key}(n) < \text{keys}(T_R) < \text{key}(p)$$



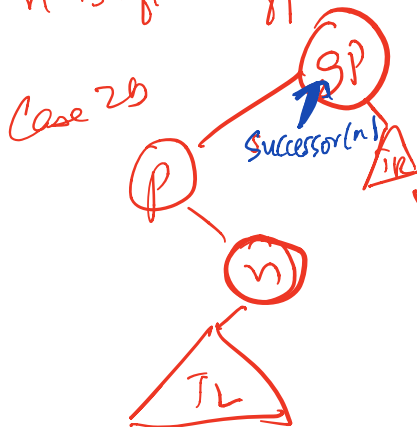
Therefore Successor⁽ⁿ⁾ must be in T_R

Case 2: n has no right subtree (this was left as an assignment) but now that you know about the Inorder Traversal, the proof might be easier to follow



n is left child of parent

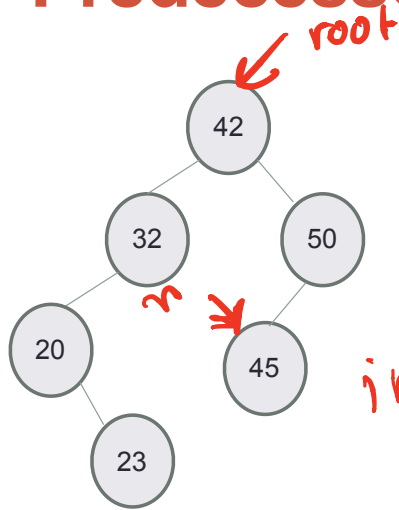
If n is the left child of its parent (as shown on the figure to the left), an inorder traversal of the tree would print the parent's key after printing n . Therefore p is the successor of n



n is right child of parent

If n is the right child of its parent (p) then, the parent's recursive call is done after n 's key is printed and we proceed to the grandparent's recursive call OR more generally proceed all the way up the tree until we find a node that is the left child of its parent, and return that node.

Predecessor: Next smallest element



- What is the predecessor of 32?
- What is the predecessor of 45?

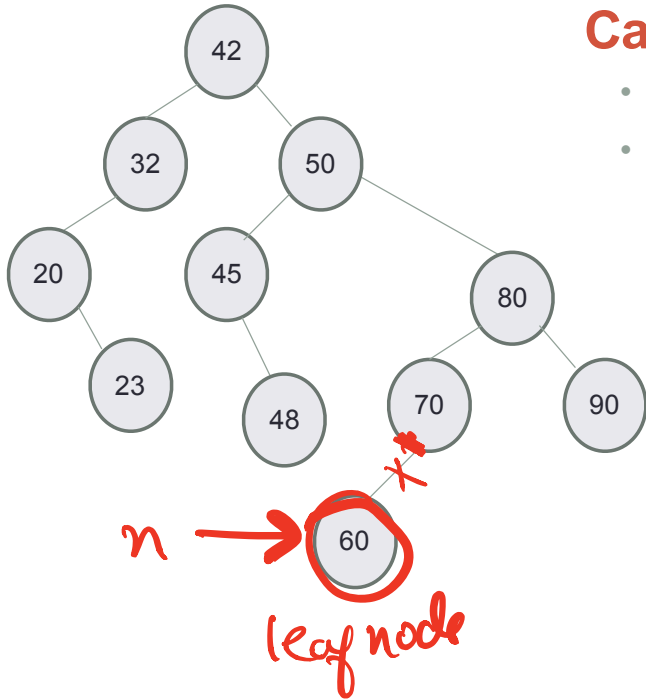
int

}
Node*

}

```
predecessor ( int value ) {  
Node * n = findH( value );  
return predecessor H ( n ) -> data ;  
}  
predecessor H ( Node * n ) {
```

Delete: Case 1

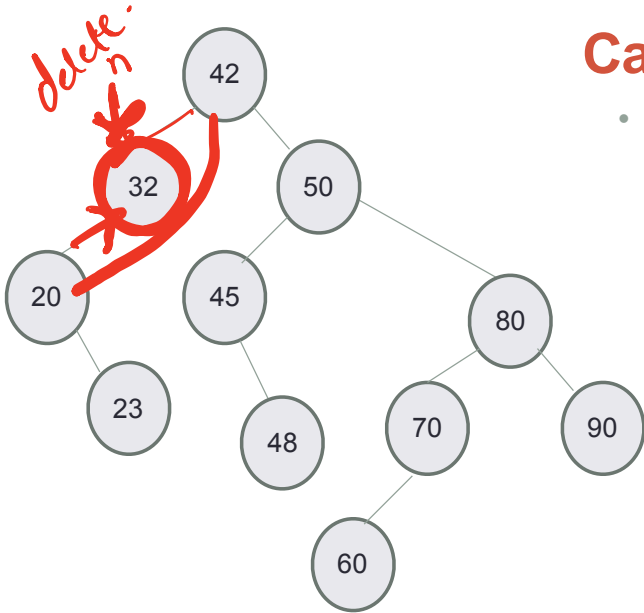


Case 1: Node is a leaf node

- Set parent's (left/right) child pointer to null
- Delete the node

```
if (!n->left && !n->right) {  
    // leaf node  
    if (n == n->parent->left)  
        n->parent->left = 0;  
    else  
        n->parent->right = 0;  
    delete n;  
}
```


Delete: Case 2

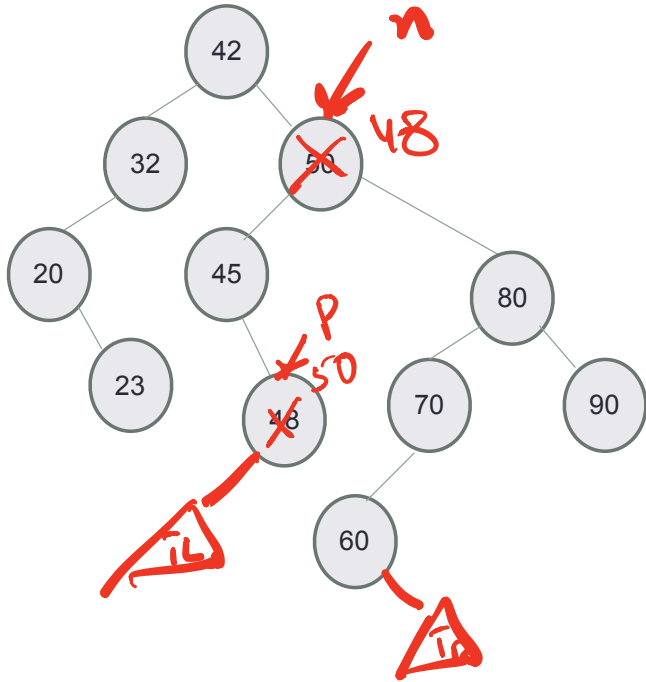


Case 2 Node has only one child

- Replace the node by its only child

```
if ( !n->right && n->parent )  
    // connect n's left child to n's parent  
    n->left->parent = n->parent;  
if ( n == n->parent->left )  
    n->parent->left = n->left;  
else  
    n->parent->right = n->left;  
// other  
delete n;
```

Delete: Case 3



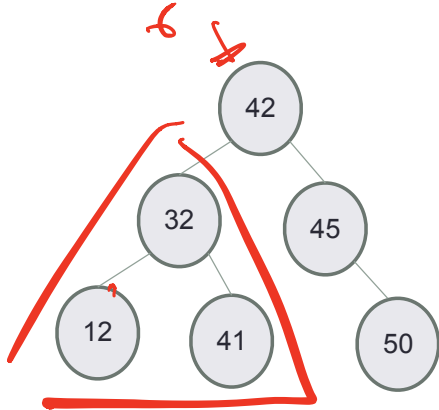
Case 3 Node has two children

- Can we still replace the node by one of its children? Why or Why not?

// swap $n \rightarrow$ data with its
predecessor OR
its successor

// delete the predecessor
// Default to either case 1
or case 2

In order traversal: print elements in sorted order



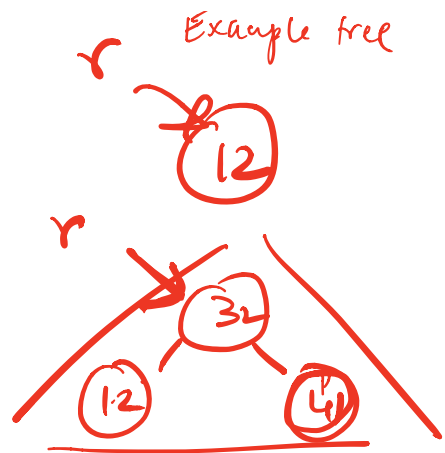
Algorithm Inorder(tree)

1. Traverse the left subtree, i.e., call Inorder(left-subtree)
2. Visit the root.
3. Traverse the right subtree, i.e., call Inorder(right-subtree)

What is the output of doing an in order traversal on the above tree

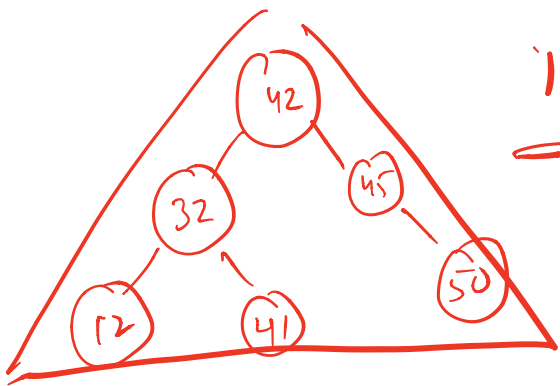
- A. 32 12 41 42 45 50
- B. 12 32 41 42 45 50
- C. 12 32 42 45 41 50
- D. None of the above

```
void InOrder (Node* r) {  
    if (!r) return  
    InOrder (r->left);  
    cout << r->data;  
    InOrder (r->right);  
}
```



Output of In Order Traversal
12

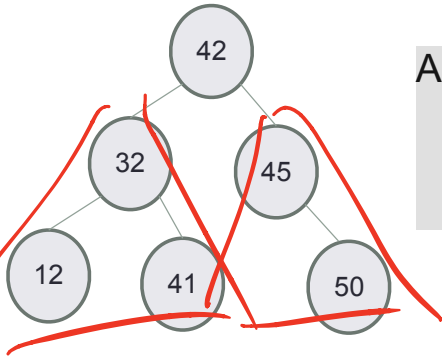
12 32 41



12 32 41 42 45 50

Keys are printed in sorted order

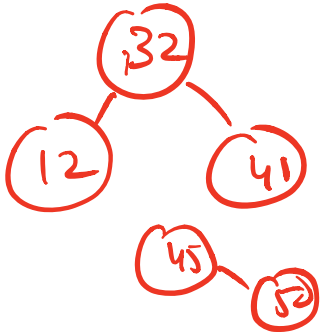
Pre-order traversal: nice way to linearize your tree!



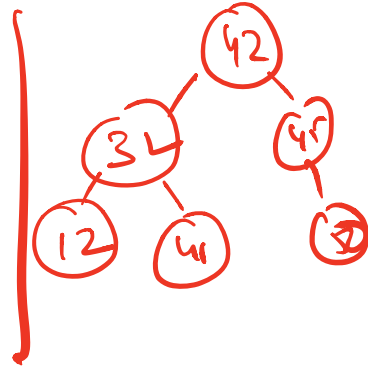
Algorithm Preorder(tree)

1. Visit the root.
2. Traverse the left subtree, i.e., call Preorder(left-subtree)
3. Traverse the right subtree, i.e., call Preorder(right-subtree)

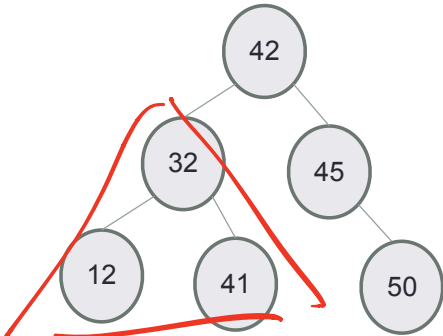
42 32 12 41 45 50



32 12 41
45 50



Post-order traversal: use in recursive destructors!



12 41 32

Algorithm Postorder(tree)

1. Traverse the left subtree, i.e., call Postorder(left-subtree)
2. Traverse the right subtree, i.e., call Postorder(right-subtree)
3. Visit the root.

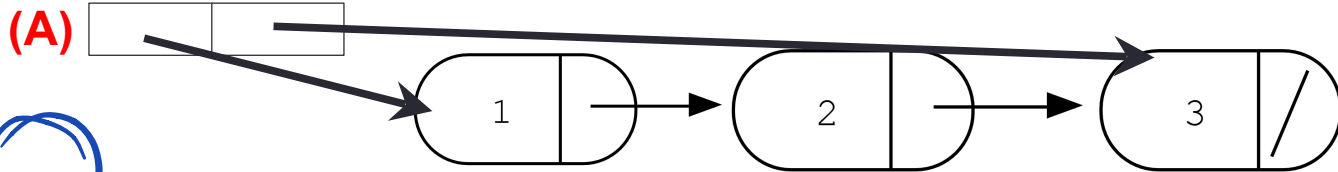
Concept Question

```
LinkedList::~~LinkedList(){  
    delete head;  
}
```

```
class Node {  
    public:  
        int info;  
        Node *next;  
};
```

Which of the following objects are deleted when the destructor of Linked-list is called?

head tail



(B) only the first node

(C): A and B

(D): All the nodes of the linked list

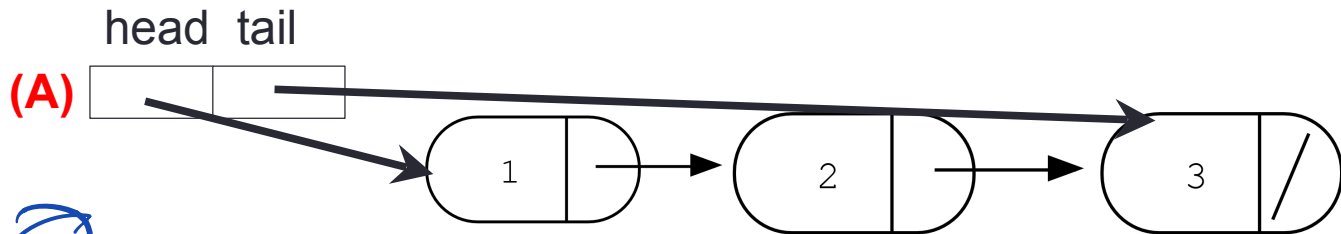
(E): A and D

Concept Question

```
LinkedList::~~LinkedList(){  
    delete head;  
}
```

```
Node::~~Node(){  
    delete next;  
}
```

Which of the following objects are deleted when the destructor of Linked-list is called?



(B): All the nodes in the linked-list

(C): A and B

(D): Program crashes with a segmentation fault

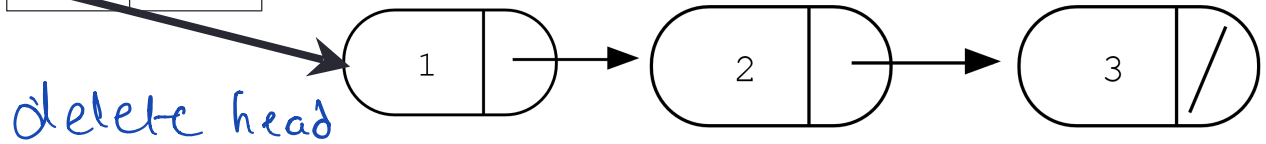
(E): None of the above


```
LinkedList::~~LinkedList(){
    delete head; // calls the first node's
                 // destructor
}
```

```
Node::~~Node(){
    delete next; // calls the
                // next node's destructor
}
```



① delete next → ② delete next → ③ delete 0



↑ does not
s of fault

call to destructor completes,
Node - 3 is delete

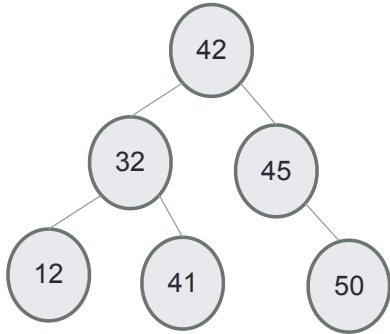
←
call to destructor of 2 completes

Node 2 is deleted

←
call to destructor of node 1 completes,
Node 1 is deleted

ll is
deleted ←

Post-order traversal: use in recursive destructors!



Algorithm Postorder(tree)

1. Traverse the left subtree, i.e., call Postorder(left-subtree)
2. Traverse the right subtree, i.e., call Postorder(right-subtree)
3. Visit the root.

in BSTNode () {

delete left;

delete right;

}