HEAPS

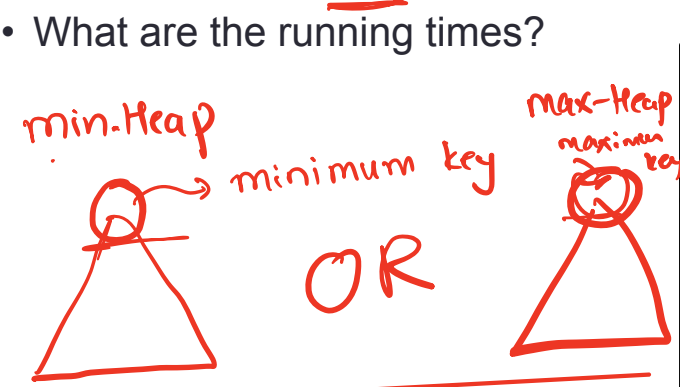
Problem Solving with Computers-II

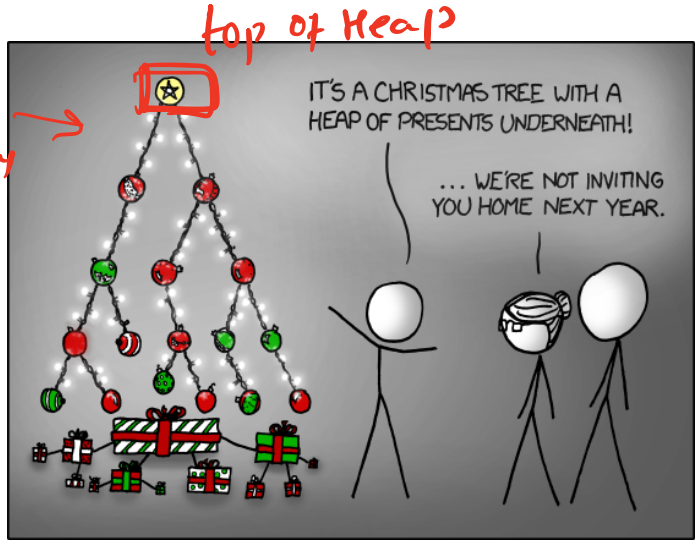
Reminders

- PA02 released, due Friday of Week 10 .
- Lab07 due Wed of Week 10
- zyBook, Chapter 8 activities due Monday of Finals week

Heaps (Priority Queue)

- Clarification
  - *heap*, the data structure is not related to *heap*, the region of memory
- What are the operations supported?
- What are the running times?





min-heap

O(1)	min
O(logN)	insert
O(logN)	delete min
O(1)	

max-heap

max	
insert	
delete-max	

STL (PQ)

top()
push()
pop()
empty()

Heaps

	Min-Heaps	Max-Heap	BST
• Insert :			
• Min:			
• Delete Min:			
• Max			
• Delete Max			

Applications:

- Efficient sort
- Finding the median of a sequence of numbers
- Compression codes

Choose heap if you are doing repeated insert/delete/(min OR max) operations

std::priority_queue (STL's version of heap)

A C++ `priority_queue` is a generic container, and can store any data type on which an ordering can be defined: for example `ints, structs (Card),` pointers etc.

```
#include <queue>
priority_queue<int> pq;
```

Methods:

```
* push()  //insert
* pop()   //delete max priority item
* top()   //get max priority item
* empty()  //returns true if the priority queue is empty
* size() //returns the number of elements in the PQ
```

- You can extract object of highest priority in O(log N)
- To determine priority: objects in a priority queue must be comparable to each other

STL Heap implementation: Priority Queues in C++

What is the output of this code?

```
priority_queue<int> pq;
pq.push(10);
pq.push(2);
pq.push(80);
cout<<pq.top();
pq.pop();
cout<<pq.top();
pq.pop();
cout<<pq.top();
pq.pop();
```

A. 10 2 80
B. 2 10 80
C. 80 10 2
D. 80 2 10
E. None of the above

std::priority_queue template arguments

```
template <
      class T,
      class Container= vector<T>,
      class Compare = less <T>
      > class priority_queue;
```

The template for priority_queue takes 3 arguments:

1. Type elements contained in the queue.
2. Container class used as the internal store for the priority_queue, the default is **vector<T>**
3. Class that provides priority comparisons, the default is **less**

std::priority_queue template arguments

```
//Template parameters for a max-heap
priority_queue<int, vector<int>, std::less<int>> pq;

//Template parameters for a min-heap
priority_queue<int, vector<int>, std::greater<int>> pq;
```

Comparison class

- Comparison class: A class that implements a function operator for comparing objects

```
class compareClass{
      bool operator()(int& a, int & b) const {
            return a>b;
       }
};
```

Comparison class

```
Class compareClass{
      bool operator()(int& a, int & b) const {
            return a>b;
       }
};
```

```
int main(){
    compareClass c;
    cout<<c(10, 20)<<endl;
}
```

What is the output of this code?

A. 1

B. 0

C. Error

STL Heap implementation: Priority Queues in C++

```
class cmp{
        bool operator()(int& a, int & b) const {
                return a>b;
         }
};
 priority_queue<int, vector<int>, cmp> pq;
 pq.push(10);
 pq.push(2);
 pq.push(80);
 cout<<pq.top();
 pq.pop();
 cout<<pq.top();
 pq.pop();
 cout<<pq.top();
 pq.pop();
```

Output: _____

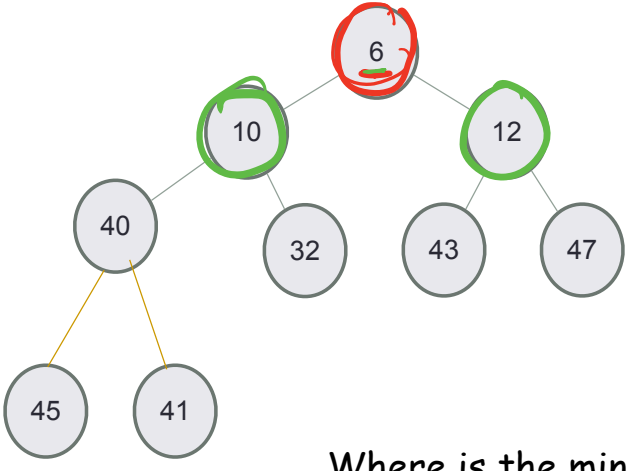
pq is a _____heap

Heaps as binary trees

- **Rooted binary tree that is as complete as possible**
- **In a min-Heap, each node satisfies the following heap property:**

key(x)<= key(children of x)

Min Heap with 9 nodes

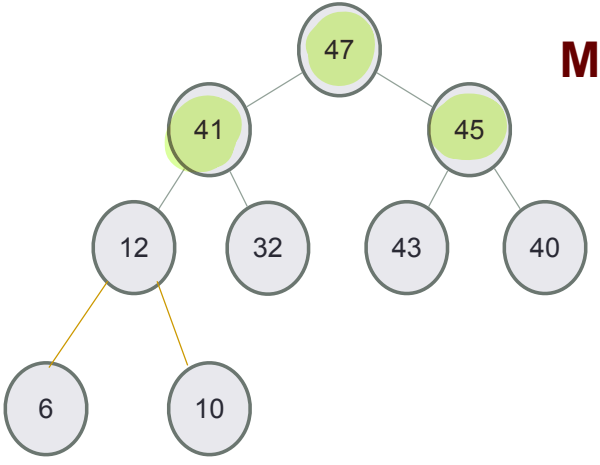


Where is the minimum element?

Heaps as binary trees

- **Rooted binary tree that is as complete as possible**
- **In a max-Heap, each node satisfies the following heap property:**

  key(x)>= key(children of x)



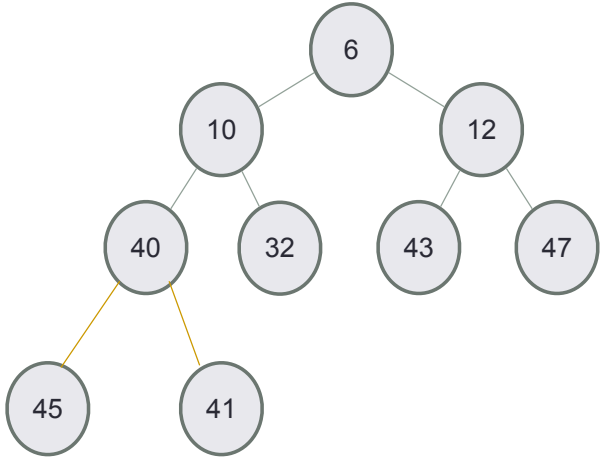
Max Heap with 9 nodes

Where is the maximum element?

Structure: Complete binary tree

A heap is a complete binary tree: Each level is as full as possible.
Nodes on the bottom level are placed as far left as possible

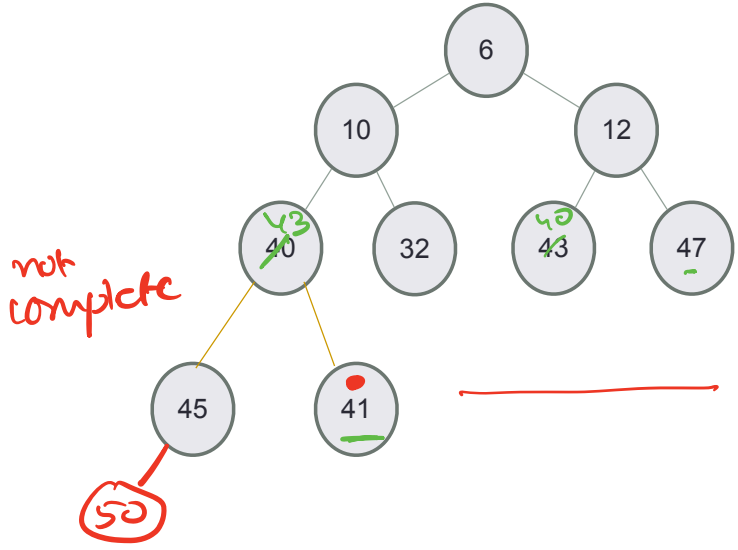
$H = O(\log N)$



Identifying heaps

Starting with the following min-Heap which of the following operations will result in something that is NOT a min Heap

A. Swap the nodes 40 and 32
B. Swap the nodes 32 and 43
C. Swap the nodes 43 and 40
D. Insert 50 as the left child of 45
E. C&D

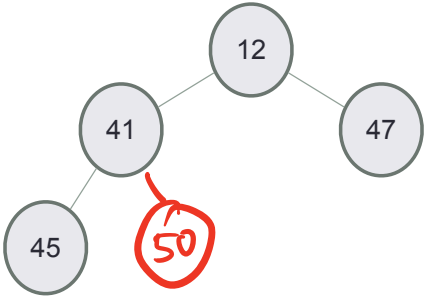


Insert 50 into a heap

push (50)

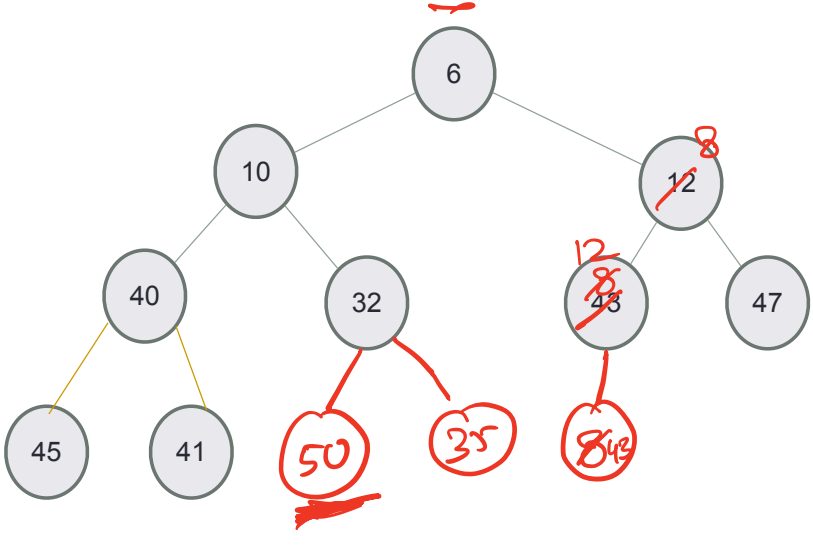
- Insert key(x) in the first open slot at the last level of tree (going from left to right)
- If the heap property is not violated - Done
- Else: while(key(parent(x))>key(x)) swap the key(x) with key(parent(x))

min-heap



Insert 50 as the right child of 41 : O(1)

Insert 50, then 35, then 8



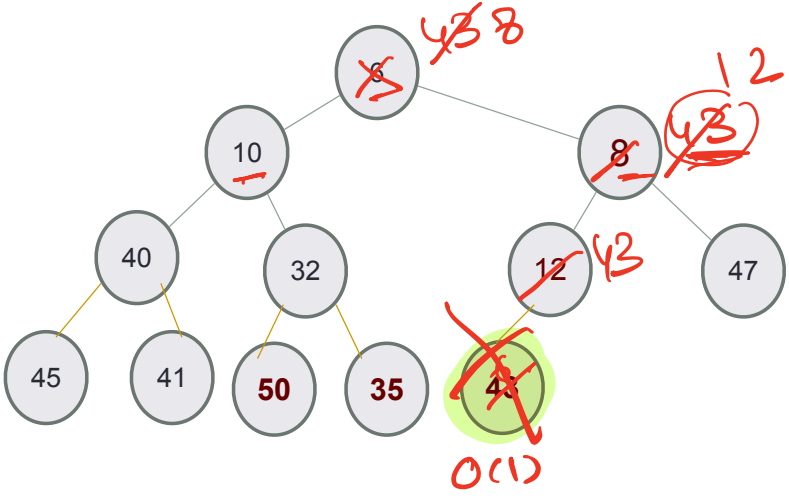
"Bubble up"

push : O(log N)

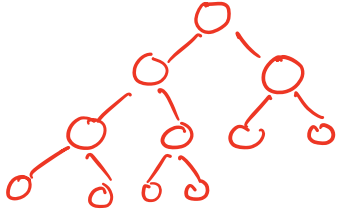
Delete min

pop() : delete the "top" key

- **Replace the root with the rightmost node at the last level**
- **"Bubble down"- swap node with child with the smallest key value until the heap property is restored**

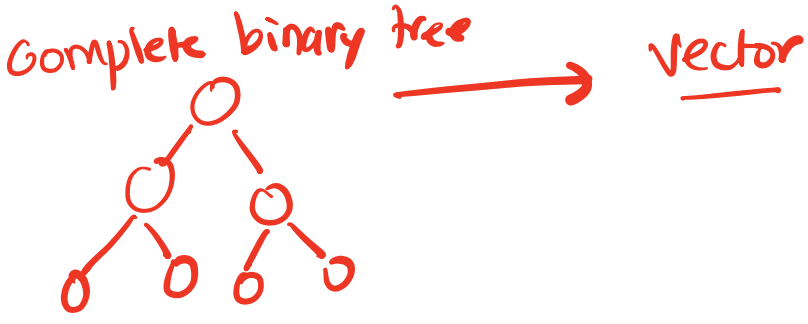


"Bubble down"



Under the hood of heaps

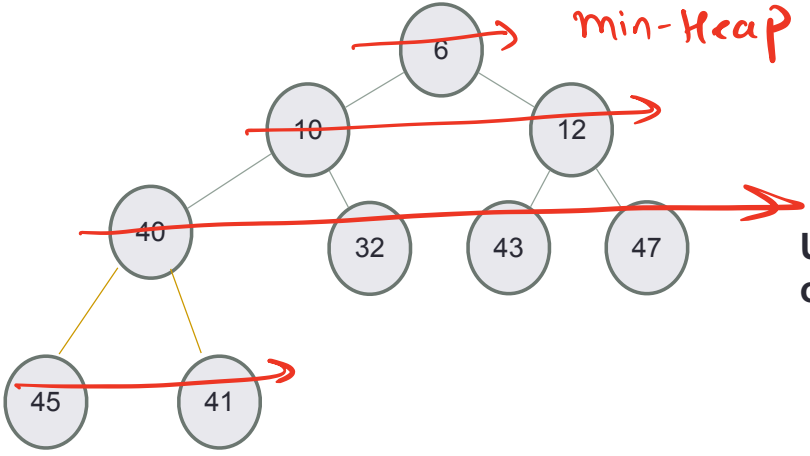
- An efficient way of implementing heaps is using vectors
- Although we think of heaps as trees, the entire tree can be efficiently represented as a vector!!



Implementing heaps using an array or vector

top()
push()
pop()
empty()

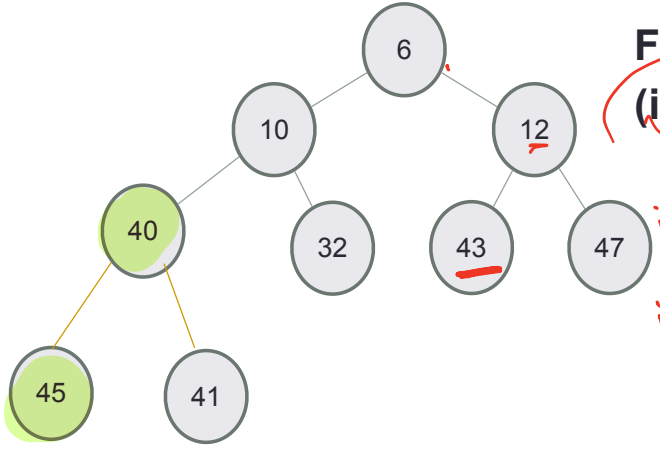
Value	6	10	12	40	32	43	47	45	41	
Index	0	1	2	3	4	5	6	7	8	9



Using vector as the internal data structure of the heap has some advantages:

- **More space efficient than trees**
- **Easier to insert nodes into the heap**

Finding the "parent" of a "node" in the vector representation



For a key at index i, index of the parent is (i-1)/2

index_parent (i) = $\left(\frac{i-1}{2}\right)$

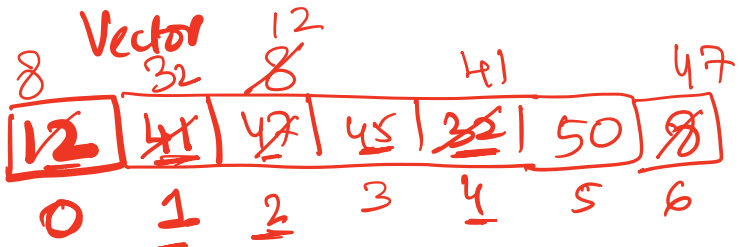
index_parent (index=5) = 2

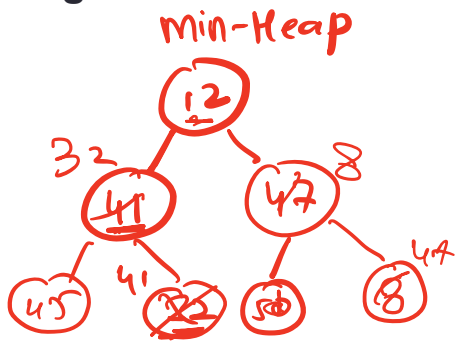
Value	6	10	12	40	32	43	47	45	41
Index	0	1	2	3	4	5	6	7	8
Index of parent	–	0	0	1	1	2	2	3	3

Insert into a heap

- Insert key(x) in the first open slot at the last level of tree (going from left to right)
- If the heap property is not violated - Done
- Else….

Insert the elements {12, 41, 47, 45, 32} in a min-Heap using the vector representation of the heap



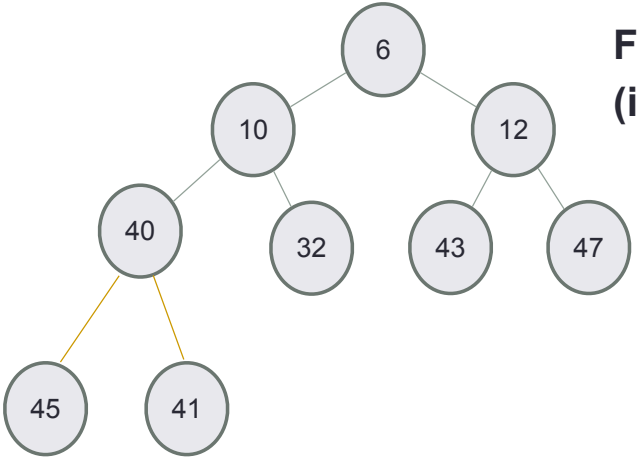


$\left\lfloor \frac{0-1}{2} \right\rfloor = -1$

$\frac{1-1}{2} = 0$

Find the index of the parent of 32 in the vector

Insert 50, then 35



For a node at index i, index of the parent is (i-1)/2

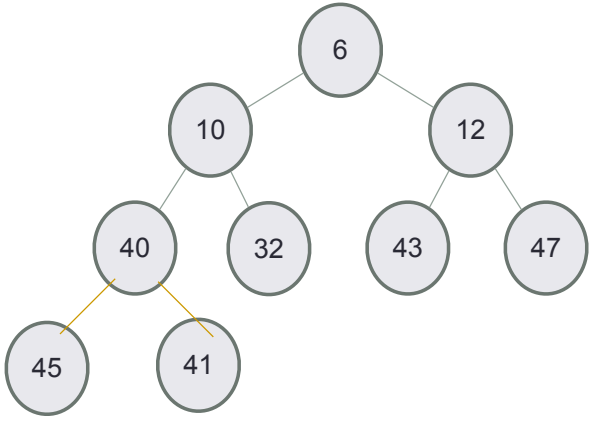
Value	6	10	12	40	32	43	47	45	41	
Index	0	1	2	3	4	5	6	7	8	

Traversing down the tree

pop(): delete min

Value	6	10	12	40	32	43	47	45	41	
Index	0	1	2	3	4	5	6	7	8	

Left child (i) = 2i + 1
Right child (i) = 2i + 2



For a node at index i, what is the index of the left and right children?

A. (2*i, 2*i+1)

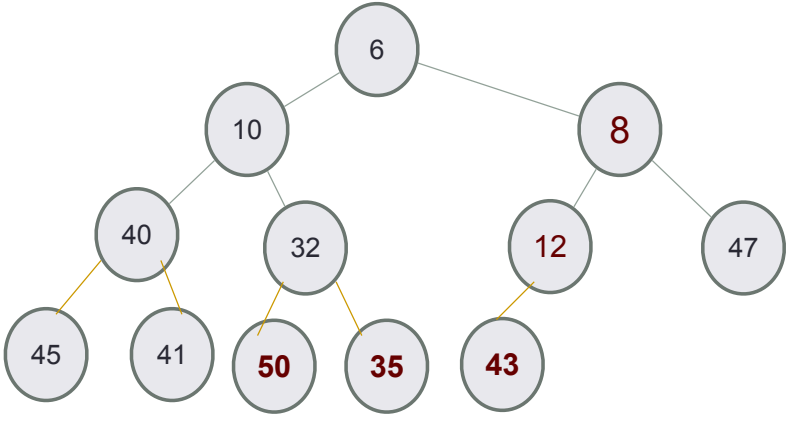
B. (2*i+1, 2*i+2)

C. (log(i), log(i)+1)

D. None of the above

Delete min

- **Replace the root with the rightmost node at the last level**
- **“Bubble down”- swap node with one of the children until the heap property is restored**



Insert 8 into a heap

Value	6	10	12	40	32	43	47	45	41	50	35
Index	0	1	2	3	4	5	6	7	8	9	10

After inserting 8, which node is the parent of 8 ?

A. Node 6

B. Node 12

C. None 43

D. None - Node 8 will be the root