

# FINAL PRACTICE

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# Final exam: In person

Time: Tuesday March 15, noon - 3p

Location: BUCHN 1910

Read the instructions for the exam carefully:

→ <https://ucsb-cs24.github.io/w22/exam/e02/> ✖

- ★ 1 page of notes is okay.
- ★ Seating is assigned
- ★ State your assumptions
- ★ Bring your IDs.
- ★ Dark pencil or pen

- input size  $T(n) = O(n^3)$
- The runtime complexity of an algorithm is  $T(n) = 5n^2 \log n + n + 1000$
  - Show that  $T(n) = \theta(n^2 \log n)$  using the definition of Big-Theta

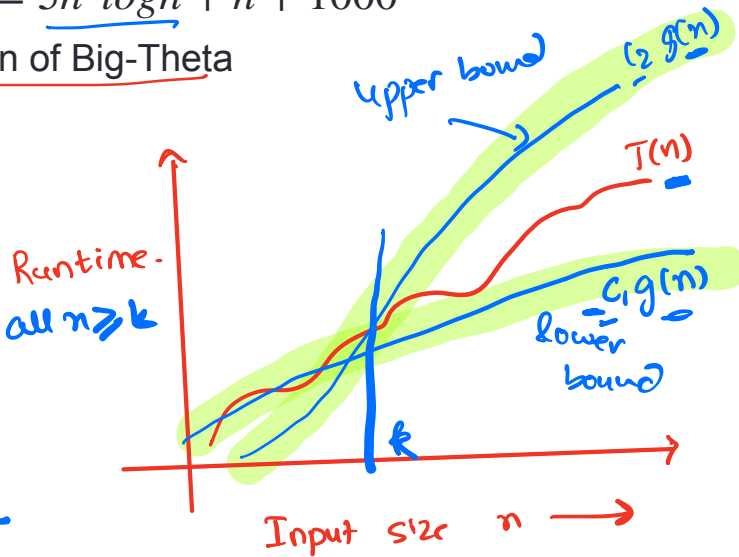
$$T(n) = \Theta(g(n)) \\ = O(g(n))$$

$$c_1 > 0, c_2 > 0, g(n) \geq 0, k \geq 0 \\ c_2 g(n) \leq T(n) \leq c_1 g(n), \text{ for all } n \geq k$$

$$\underline{5n^2 \log n} + \underline{n} + \underline{1000}$$

$$n \leq n^2 \log n, \text{ for } n \geq 2$$

$$1000 \leq \frac{10}{n^2 \log n}, \text{ for } n \geq 10$$



therefore

$$T(n) \leq 5n^2 \log n + n^2 \log n + 10n^2 \log n$$

$$= \underbrace{16}_{c_2} n^2 \log n \quad n \geq 10$$

- (1)

$$\underbrace{3}_{c_1} n^2 \log n \leq \underbrace{5n^2 \log n}_{-} + \cancel{n} + \frac{1000}{-}, n \geq 10$$

- (2)

$$T(n) = \Theta(n^2 \log n)$$

$c_1 = 3 \quad c_2 = 16 \quad k = 10$

- Assume **dataA** is some data structure and the input vector **v** has **N** key
- Describe algoX in a sentence

```
void algoX(vector<int>& v)
{
    dataA ds; // empty
    for(auto& elem: v)
        ds.insert(elem);
    for(auto& elem: v){
        elem = ds.min();
        ds.delete(elem);
    }
}
```

- Assume **dataA** is some data structure and the input vector to algoX has N numbers
- Given: running time of operations for **dataA**, where **M** is the number of keys stored in dataA

worst case  
 {  
 • insert:  $O(\log M)$   
 • min:  $O(1)$   
 • delete:  $O(\log M)$   
 }

← make use to upper bound

↓  
 no. of keys already stored in the data structure.

```
void algoX(vector<int>& v)
```

```
{
```

$O(1)$

```
    dataA ds; // empty
```

```
    for(auto& elem: v)
```

```
        ds.insert(elem);
```

```
    for(auto& elem: v){
```

```
        elem = ds.min();
```

```
        ds.delete(elem);
```

```
    }
```

```
}
```

What is the Big-O running time of algoX?

A.  $O(N^2)$

B.  $O(N \log N)$

C.  $O(N)$

D.  $O(\log N)$

E. Not enough information to compute

N-1 keys

$c_1 < c_2 < c_3 < \dots < c_n$   
 → maximum time  $O(\log N)$

ds.insert(elem) takes a variable time in each iteration but we can upper bound it by the time it takes to do the last insert

$$\text{ds.insert(elem)} \leq \text{ds.insert(last elem)} = O(\log N)$$


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First delete will take the most time because ds has the most keys  $\leq (N)$ . Use that to upper bound the runtime of all subsequent deletions.

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$$T(n) = \underbrace{O(1)}_{\text{ds.}} + \underbrace{n}_{\text{delete}} \cdot O(\log n) + \underbrace{n}_{\text{min}} (O(\log n) + O(1))$$

$$= O(n \log n)$$

$$T(n) = \underline{n^2 m} + \underline{n^2 \log m}$$

$$= O(n^2 m)$$

$$O(\underline{n^2 m} + \underline{n \cdot m \cdot k^3})$$

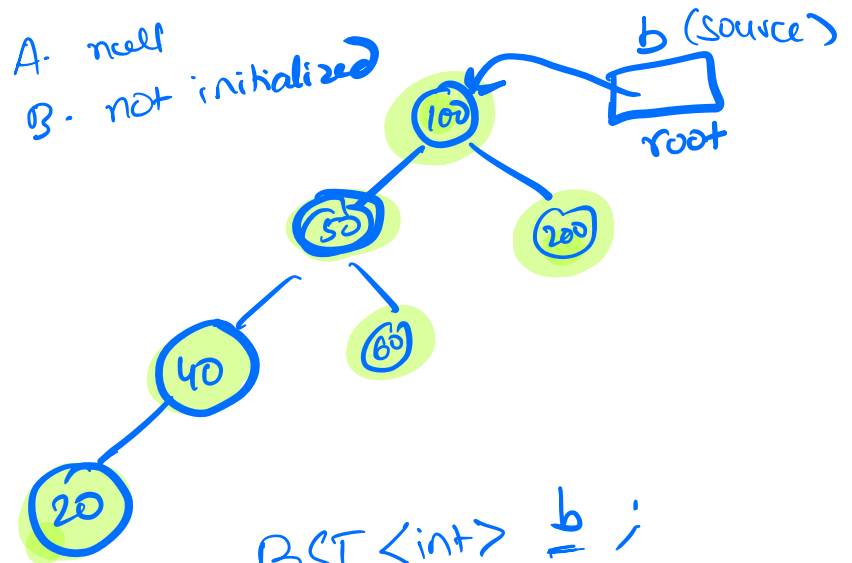
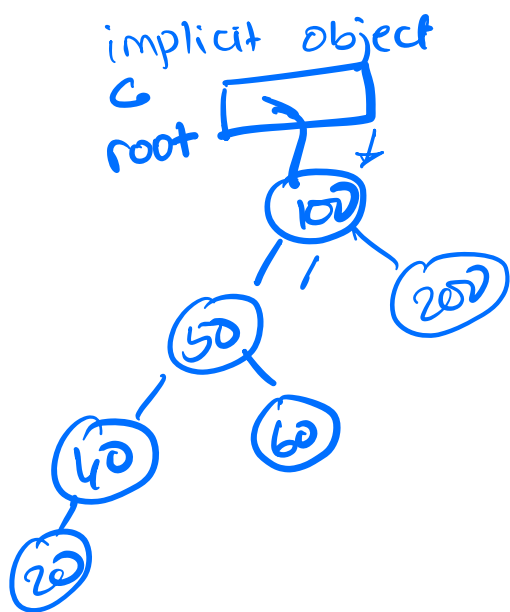
## Big Four and the Rule of Three

Implement the copy constructor for a BST

~~Big Four~~  
Constructor  
Destructor ✓  
→ copy-constructor  
copy-assignment operator } compiler will provide a default.

- A. Very confident
- B. Need to think!
- C. I just don't know!





```

template <class T>
class BST {

```

```

    BST<int> b ;
    // insert keys

```

```

    BST<int> c(b) ;

```

```

    BST(const BST<T> & source) {

```

```

        root = null ptr;

```

```

        if (!source.root) return;

```

```

        // Pre Order Insertion

```

```

    }
};

```

# Data structure Comparison

	Insert	Search	Min	Max	Delete min	Delete max	Delete (any)
Sorted array <i>Vector</i>							
Unsorted array <i>Vector</i>							
Sorted linked list (assume access to both head and tail) <i>list</i>							
Unsorted linked list <i>list</i>							
Stack <i>stack</i>							
Queue <i>queue deque</i>							
BST (unbalanced)							
BST (balanced) <i>Set</i>							
Min Heap <i>priority queue</i>							
Max Heap <i>"</i>							

# Data structure Comparison

	Insert	Search	Min	Max	Delete min	Delete max	Delete (any)
Sorted array	$O(N)$	$O(\log N)$	$O(1)$	$O(1)$	$O(N)$ if ascending order, else $O(1)$	$O(1)$ if ascending, else $O(N)$	$O(\log N)$ to find, $O(N)$ to delete
Unsorted array	$O(1)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$
Sorted linked list (assume access to both head and tail)	$O(N)$	$O(N)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(N)$ to find, $O(1)$ to delete
Unsorted linked list	$O(1)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$ to find, $O(1)$ to delete
Stack	$O(1)$ - only insert to top	Not supported	Not supported	Not supported	Not supported	Not supported	$O(1)$ - Only the element on top of the stack
Queue	$O(1)$ - only to the rear of the queue	Not supported	Not supported	Not supported	Not supported	Not supported	$O(1)$ - only the element at the front of the queue
BST (unbalanced)	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$	$O(N)$
BST (balanced)	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(\log N)$	$O(\log N)$
Min Heap	$O(\log N)$	Not supported	$O(1)$	Not supported	$O(\log N)$	Not supported	$O(\log N)$
Max Heap	$O(\log N)$	Not supported	Not supported	$O(1)$	Not supported	$O(\log N)$	$O(\log N)$