WANT MAX? ASK HEAP

Problem Solving with Computers-II

Make a copy of the handout for today's lecture

https://bit.ly/cs24-lect14-handout

How is PA 2 going?

Make a copy of the handout for today's lecture
https://bit.ly/cs24-lect14-handout

A. Done!

B. On track to finish and having fun!

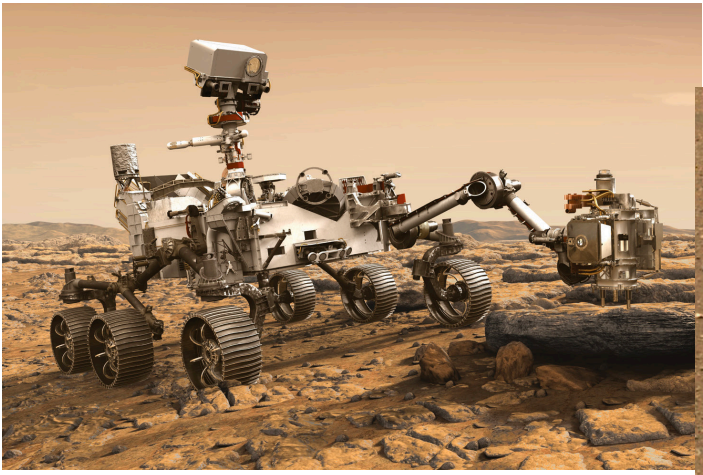
C. On track to finish but struggling (a bit).

D. Falling behind and struggling a lot

E. Haven't read the assignment.

I can deal with pressure, and deadlines.

Interview Practice!



Perseverance

landed on Mars on Feb 18, 2021

Ingenuity

Flew over Mars
on April 19, 2021

Spirit and Opportunity

Mars Exploration: 2004

What is the running time of the **inner loop**? (1 min)

```
void mystery(vector<int>& v){
    for (int i = 0; i < v.size(); i++){
        int index = i;
        for (int j = i + 1; j < v.size(); j++){
            if(v[j] > v[index]){
                index = j;
            }
        }
        if(index != i){
            int temp = v[index];
            v[index] =  v[i];
            v[i] = temp;
        }
    }
}
```

n = v.size()

find index of max v[i : (n-1)]

O(n)

What is the **inner loop** doing (in plain English)?

What is **mystery** doing ?
What is its space and time complexity?

Run Time $O(n^2)$
Space $O(1)$

(2 min)

```
void mystery(vector<int>& v){
    for (int i = 0; i < v.size(); i++){
        int index = i;
        for (int j = i + 1; j < v.size(); j++){
            if(v[j] > v[index]){
                index = j;
            }
        }
        if(index != i){
            int temp = v[index];
            v[index] =  v[i];
            v[i] = temp;
        }
    }
}
```

O(n)

find max v[i : n-1]

O(1)

i = 0
i = 1

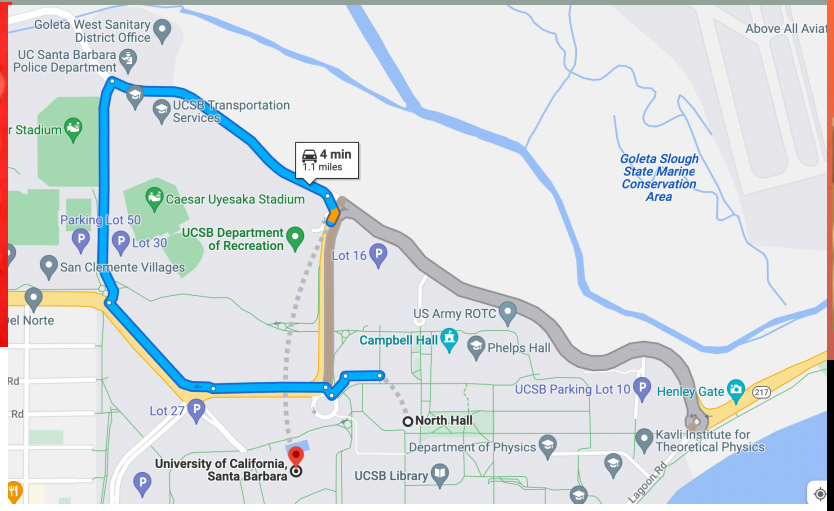
Example input:

20,	5,	7,	1,	3,	2
20	5	7	1	3	2
20	7	5	1	3	2
20	7	5	3	1	2
20	7	5	3	2	1

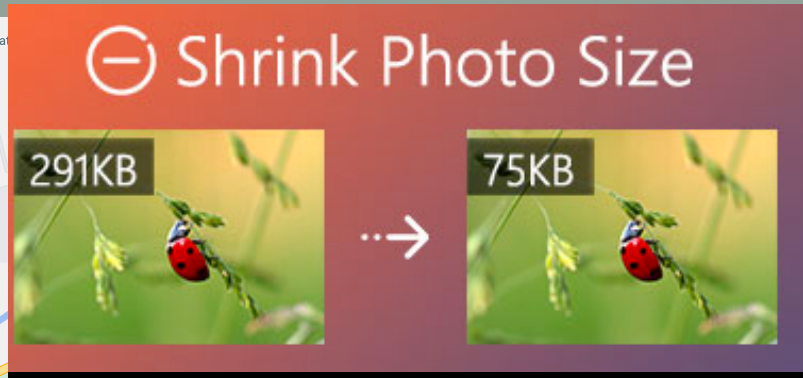
Write your ideas to improve the running time of mystery (3 min)

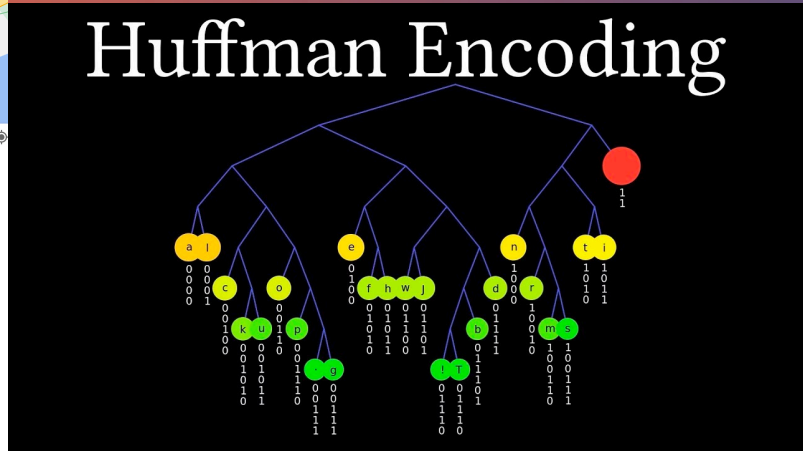
```
void mystery(vector<int>& v){
    for (int i = 0; i < v.size(); i++){
        find max of vector: v[i:n]
        swap v[i] with max element
    }
}
```

Sorting



Shortest Path





Data Compression

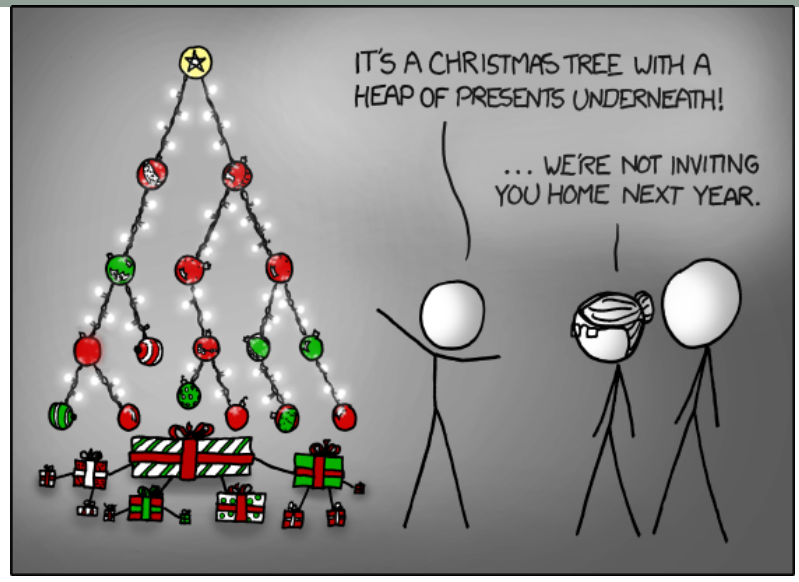
Want min or max? Ask Heap!

Many algorithms need to compute the min OR max repeatedly

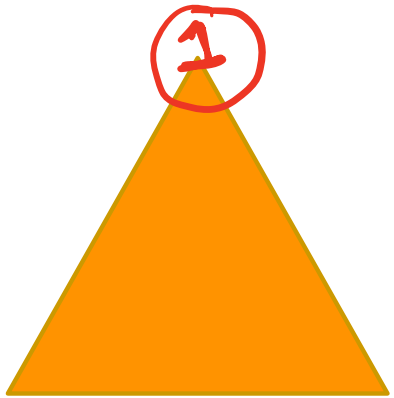
Heap is used speed up the running time of such algorithms!

New data structure: Heap

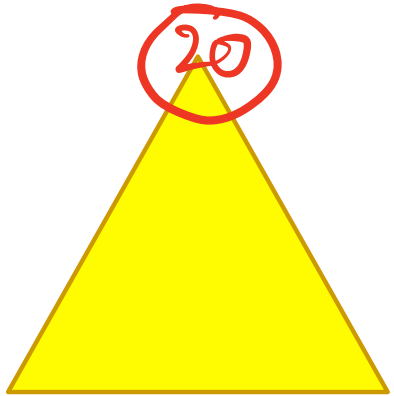
- Clarification
  - *heap*, the data structure is not related to *heap,* the region of memory
- What are the operations supported?
- What are the running times?



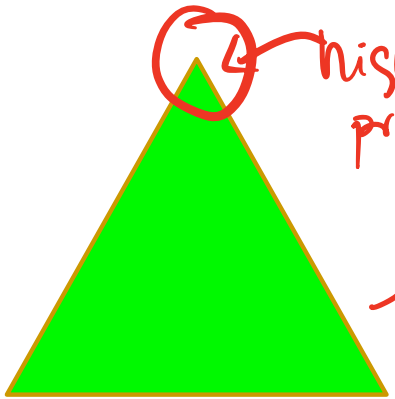
20, 5, 7, 1, 3, 2



min-heap



max-heap



push() - O(logn)
top() - O(1)
pop() - O(logn)

a > b
a has higher priority than b

std::priority_queue

Coding demo

(3 min) Now consider the interview problem again. Provide a faster algorithm to mystery using priority_queue (aka heap data structure). Analyze its space and run-time complexity.

Congrats, you got the job!

Fun fact: Ingenuity's flight control software has C++ framework that provides core capabilities.

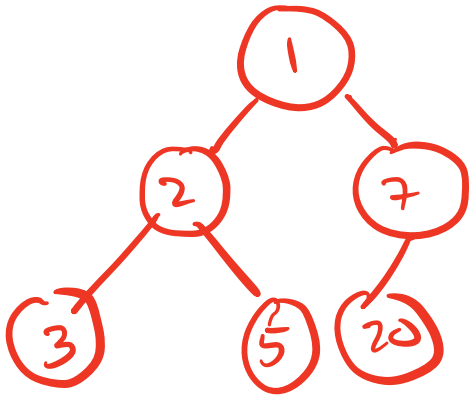
Open-Source Flight Software Framework, F-Prime: https://github.com/nasa/fprime

Reeves, Glenn E., and Joseph F. Snyder. "An overview of the Mars exploration rovers' flight software." *2005 IEEE International Conference on Systems, Man and Cybernetics*. Vol. 1. IEEE, 2005.

20, 5, 7, 1, 3, 2

min-heap

max-heap



Internally, a heap is a **complete binary tree**, where each node satisfies the heap property

In a min-heap, for each node (x):

key(x) <= key(children of x)

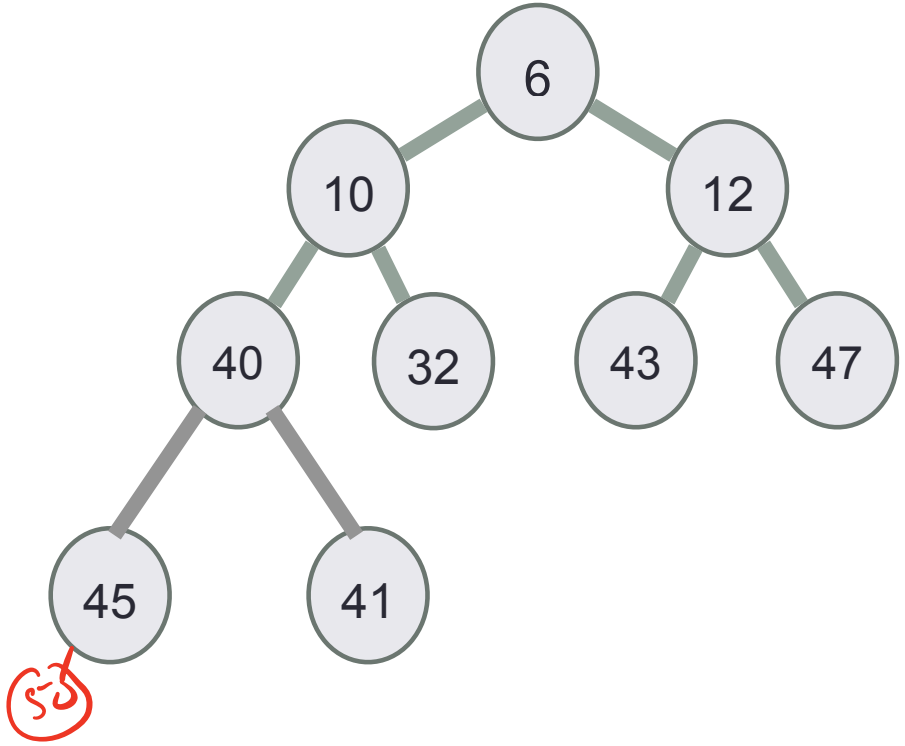
In a max-heap, for each node (x):

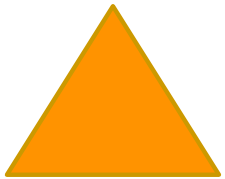
key(x) >= key(children of x)

Identifying heaps

Starting with the following min-Heap which of the following operations will result in something that is NOT a min Heap

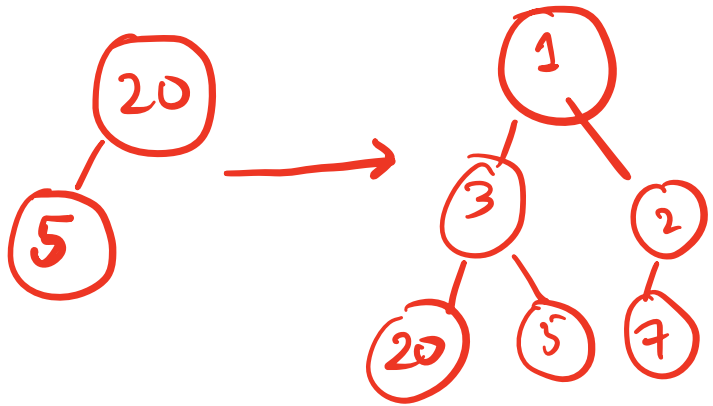
A. Swap the keys 40 and 32
B. Swap the keys 32 and 43
C. Swap the keys 43 and 40
D. Insert 50 as the left child of 45
E. C&D





min-Heap

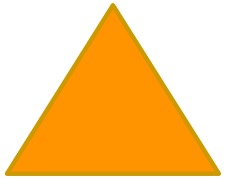
20, 5, 7, 1, 3, 2
push:O(log n)




```
procedure push(x: key value)
  insert x in the first open spot in the tree
  while(x has a parent && parent(x) > x):
    swap(x, parent(x))
  return {x was inserted into a min-heap}
```

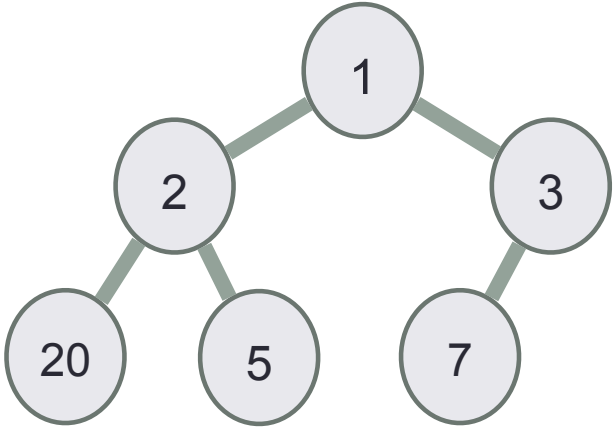
O(1)

bubble up O(log n)

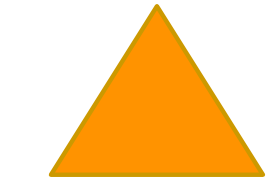


min-Heap

20, 5, 7, 1, 3, 2
top:O(1)

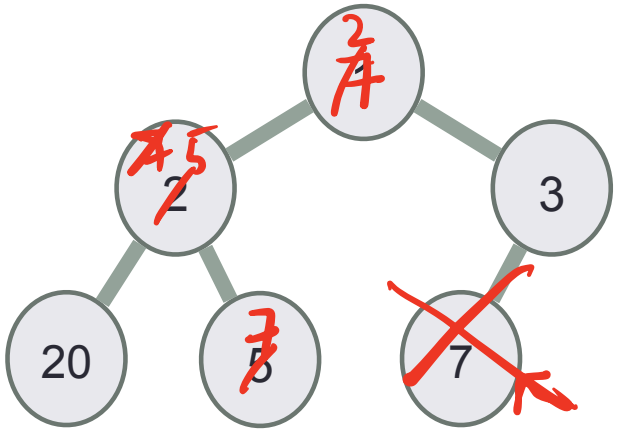



```
procedure top()
  return key of root node {top element is returned}
```



min-Heap

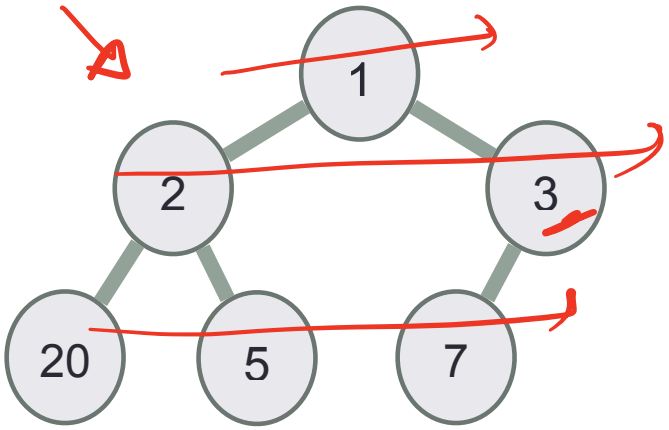
20, 5, 7, 1, 3, 2
pop:O(log n)




```
procedure pop()


  return {key on top of the heap is deleted}
```

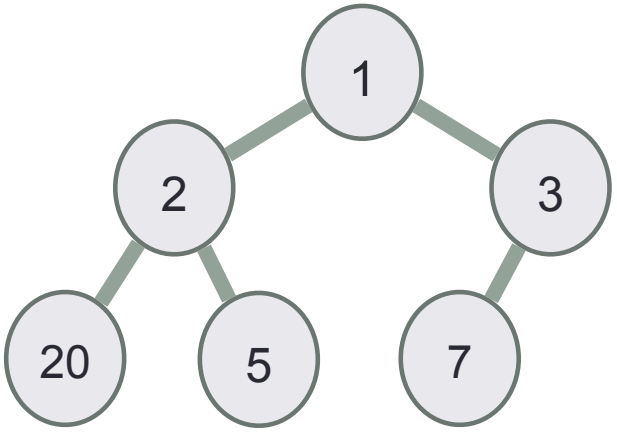
Internally the “heap binary tree” is really just a vector!

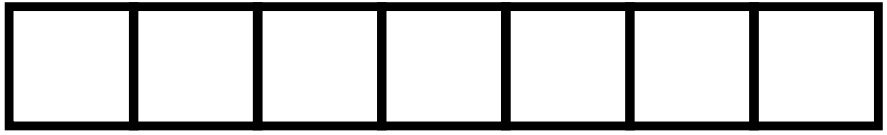


	1	2	3	20	5	7		
Index of key	0	1	2	3	4	5		
Index of parent	-1	0	0	1	1	2		
Index of left child	1	3	5	-	-	-		
Index of right child	2	4	-	-	-	-		

Work to complete the table on page 6 on your handout

Internally the “heap binary tree” is really just a vector!

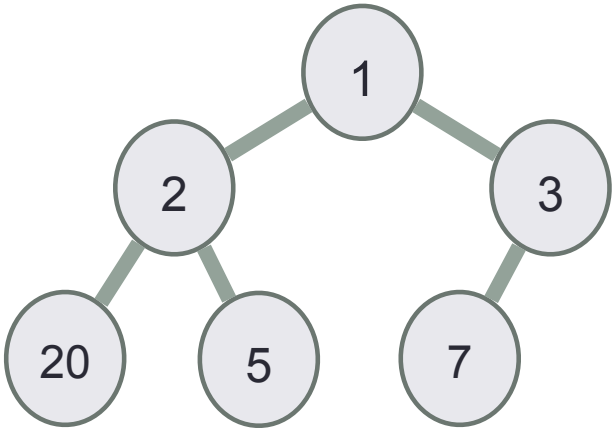


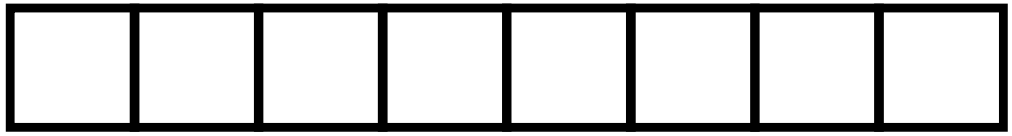


Given the index of a key in the vector, how can we find its parent?

A. Calculate the index of parent
B. Start at the root (first node in the vector), and traverse down, checking left and right children of each node until you find the parent
C. Can’t be done: A vector cannot capture parent-child relationships
D. I am not sure

Under the hood the “binary tree” is really just a vector!





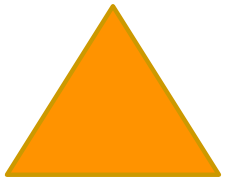
For a key at index i, what is the index of the left and right children?

A. (2i, 2i+1)

B. (2i+1, 2i+2)

C. (log(i), log(i)+1)

D. None of the above



min-Heap

20, 5, 7, 1, 3, 2
The final push!

Repeat the exercise on page 4 of your handout to insert the values 20, 5, 7, 1, 3, 2 into an initially empty min-heap. But instead of drawing the results as a tree, draw the resulting vector

```
procedure push(x: key value)
  insert x in the first open spot in the tree
  while(x has a parent && parent(x) > x):
    swap(x, parent(x))
  return
```

Next lecture

Configuring priority_queue in different ways

Queues and their applications

6 5 3 1 8 7 2 4

Heap sort: https://en.wikipedia.org/wiki/Heapsort