BINARY SEARCH TREES

Problem Solving with Computers-II

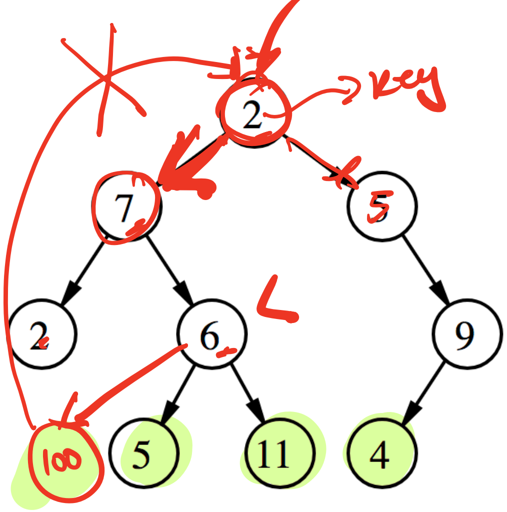
Binary Search

- Binary search. Given `value` and sorted array `a[]`, find index `i` such that `a[i]` = `value`, or report that no such index exists.
- Invariant. Algorithm maintains `a[lo]` ≤ `value` ≤ `a[hi]`.

- Ex. Binary search for 33.

6	13	14	25	33	43	51	53	64	~~72~~	~~84~~	~~93~~	~~95~~	~~96~~	~~97~~
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
lo						hi	mid							hi

Trees



A tree has following general properties:

- One node is distinguished as a **root**;
- Every node (exclude a root) is connected by a directed edge *from* exactly one other node;
  A direction is: *parent -> children*
- *Leaf node: Node that has no children*

2's children are 7 and 5

parent of 7 is 2

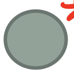
parent of 5 is 2

Which of the following is/are a tree?

empty tree

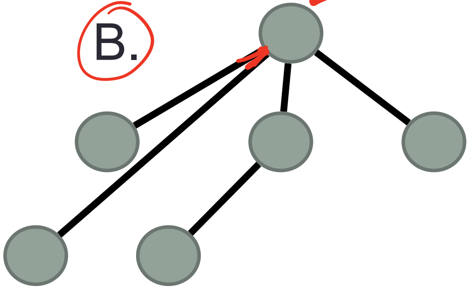
$\boxed{\text{null}}$ root

A.



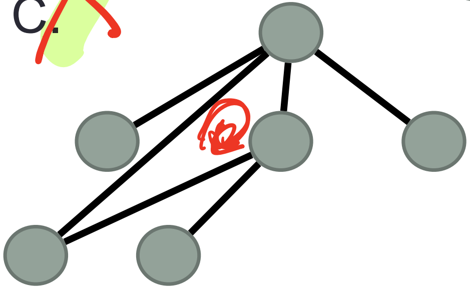
root

B.



root

C.



D. A & B

E. All of A-C

Binary Search Trees

- What are the operations supported?

  all the operations possible with sorted arrays + fast insert/delete

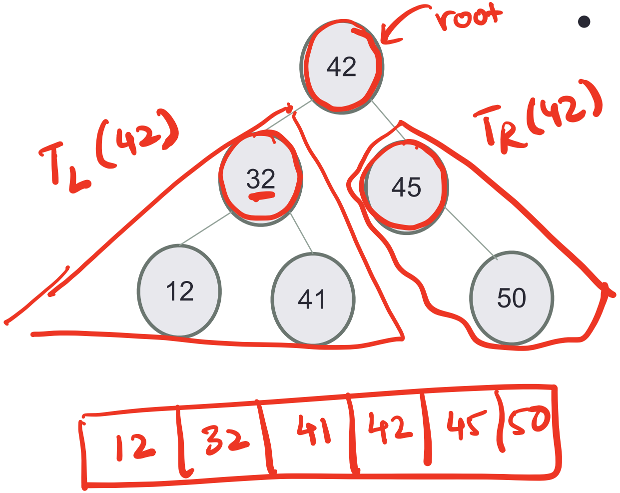
- What are the running times of these operations?

- How do you implement the BST i.e. operations supported by it?

Operations supported by Sorted arrays and Binary Search Trees (BST)

Operations	
Min	
Max	
Successor next larger value	
Predecessor next smaller value	
Search	
Insert	
Delete	
Print elements in order	

Binary Search Tree – What is it?



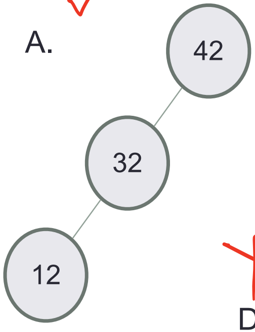
- Each node:
  - stores a key (k)
  - has a pointer to left child, right child and parent (optional)
  - Satisfies the Search Tree Property

For any node,
Keys in node's left subtree < Node's key
Node's key < Keys in node's right subtree

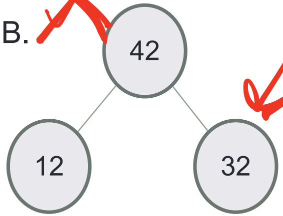
Do the keys have to be integers?

Which of the following is/are a binary search tree?

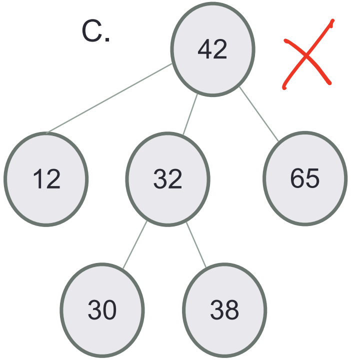
A.



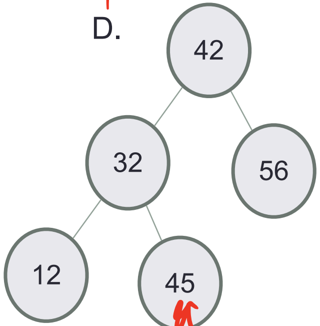
B.



C.

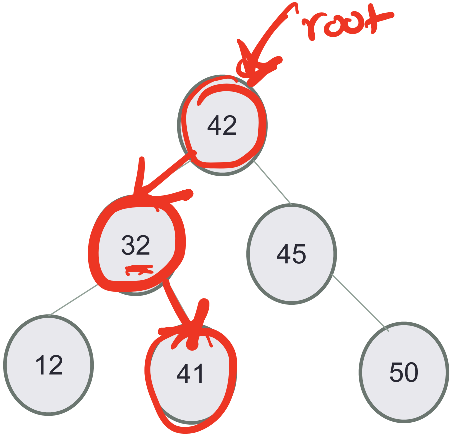


D.



E. More than one of these

BSTs allow efficient search!



- Start at the root;
- Trace down a path by comparing **k** with the key of the current node x:
  - If the keys are equal: we have found the key
  - If **k** < key[x] search in the left subtree of x
  - If **k** > key[x] search in the right subtree of x

12	32	41	42	45	50

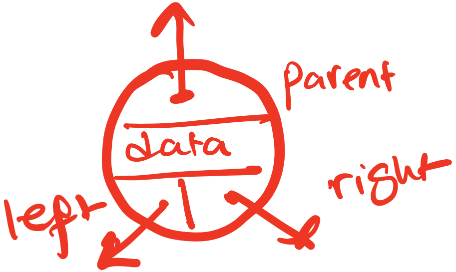
Search for 41, then search for 53

A node in a BST

```
class BSTNode {

public:
   BSTNode* left;
   BSTNode* right;
   BSTNode* parent;
   int const data;

   BSTNode( const int & d ) : data(d) {
      left = right = parent = 0;
   }
};
```

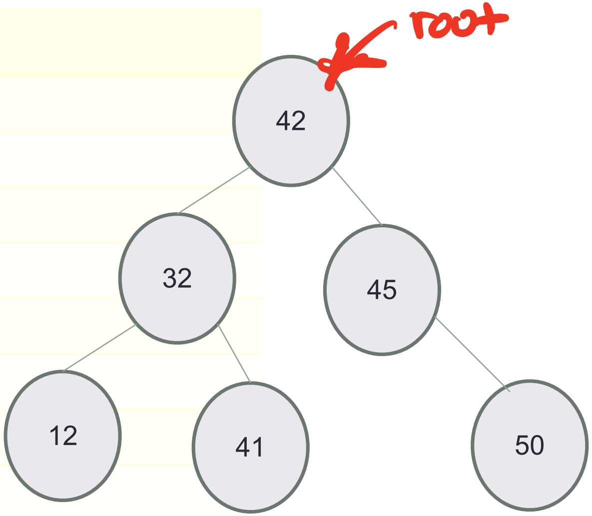


Define the BST ADT

class BST {
private:
BST Node *root;

}

Operations
Search
Insert
Min
Max
Successor
Predecessor
Delete
Print elements in order

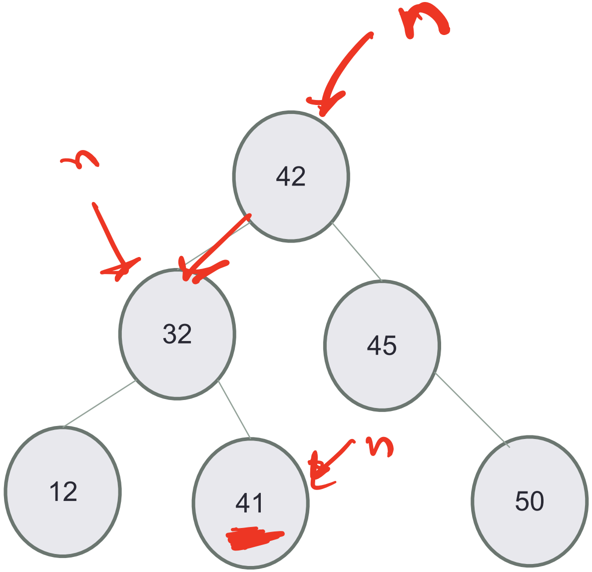


Traversing down the tree

- Suppose n is a pointer to the root. What is the output of the following code:

```
n = n->left;
n = n->right;
cout<<n->data<<endl;
```

A. 42
B. 32
C. 12
D. 41
E. Segfault

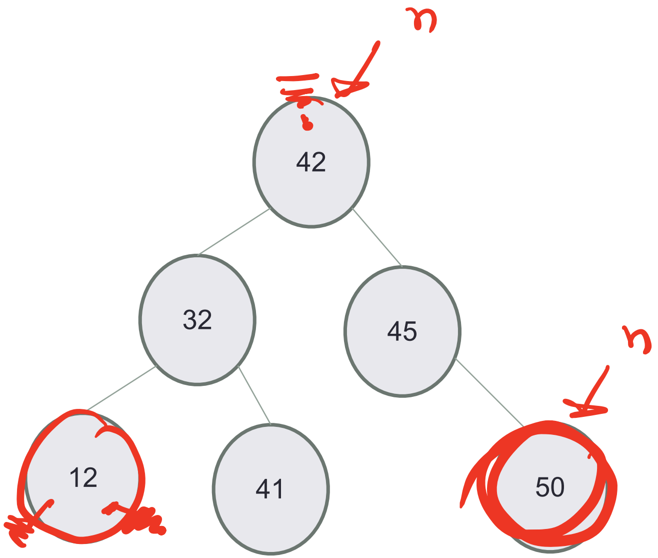


Traversing up the tree

- Suppose n is a pointer to the node with value 50.
- What is the output of the following code:

```
n = n->parent;
n = n->parent;
n = n->left;
cout<<n->data<<endl;
```

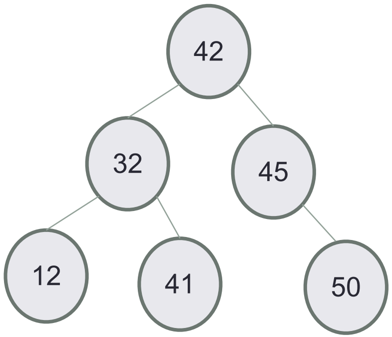
A. 42
B. 32
C. 12
D. 45
E. Segfault



Write a while loop to reach the root node, given a pointer to a node, n

```
while (n && n->parent) {
    n = n->parent;
}
```

Insert



- Insert 40
- Search for the key
- Insert at the spot you expected to find it

42, 41, 32, 12, 45, 50

Max

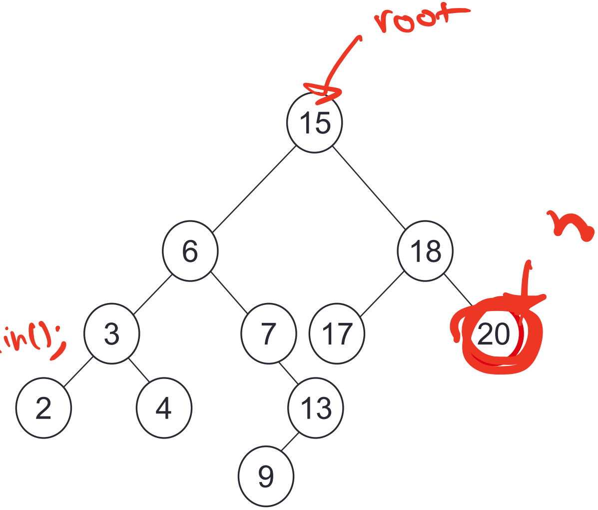
Goal: find the maximum key value in a BST

Following right child pointers from the root, until a leaf node is encountered. The least node has the max value

#include <limits.h>

Alg: **`int BST::max()`** {

```
if(!root) return std::numeric_limits<int>::min();
BSTNode *n = root;
while (n->right) {
        n=n->right;
}
return n->data;
}
```



Maximum = 20

Min

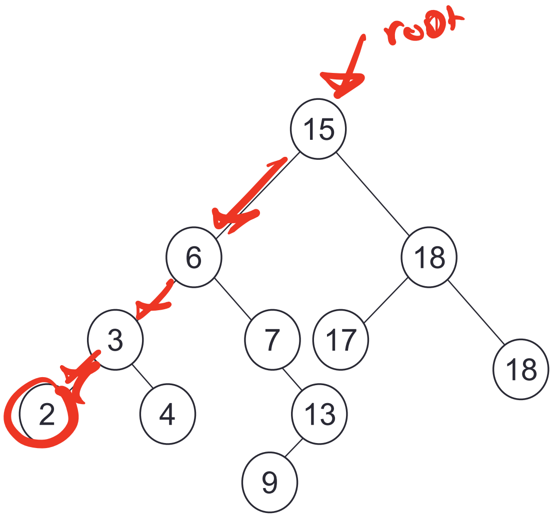
Goal: find the minimum key value in a BST

Start at the root.

Follow ________ child pointers from the root, until a leaf node is encountered

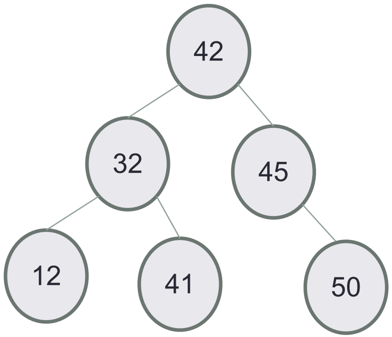
Leaf node has the min key value

Alg: `int BST::min()`



Min = ?

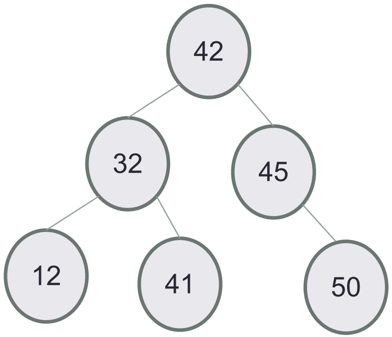
In order traversal: print elements in sorted order



Algorithm Inorder(tree)

1. Traverse the left subtree, i.e., call Inorder(left-subtree)
2. Visit the root.
3. Traverse the right subtree, i.e., call Inorder(right-subtree)

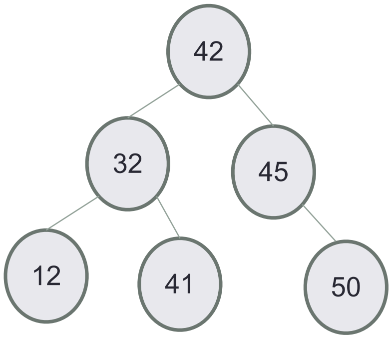
Pre-order traversal: nice way to linearize your tree!



Algorithm Preorder(tree)

1. Visit the root.
2. Traverse the left subtree, i.e., call Preorder(left-subtree)
3. Traverse the right subtree, i.e., call Preorder(right-subtree)

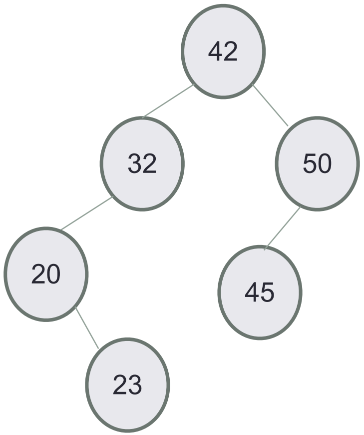
Post-order traversal: use in recursive destructors!



Algorithm Postorder(tree)

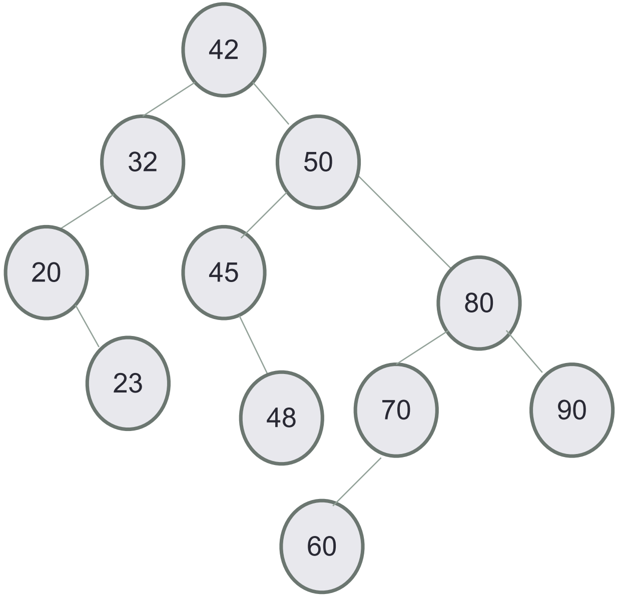
1. Traverse the left subtree, i.e., call Postorder(left-subtree)
2. Traverse the right subtree, i.e., call Postorder(right-subtree)
3. Visit the root.

Predecessor: Next smallest element



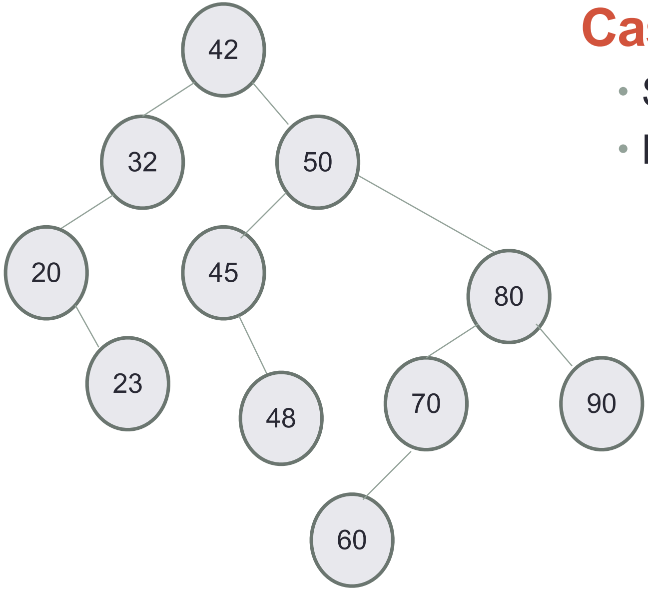
- What is the predecessor of 32?
- What is the predecessor of 45?

Successor: Next largest element



- What is the successor of 45?
- What is the successor of 50?
- What is the successor of 60?

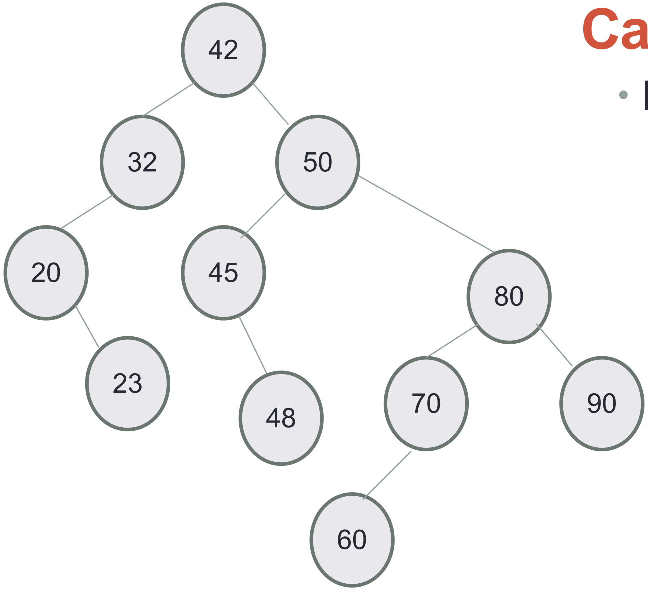
Delete: Case 1



Case 1: Node is a leaf node

- Set parent's (left/right) child pointer to null
- Delete the node

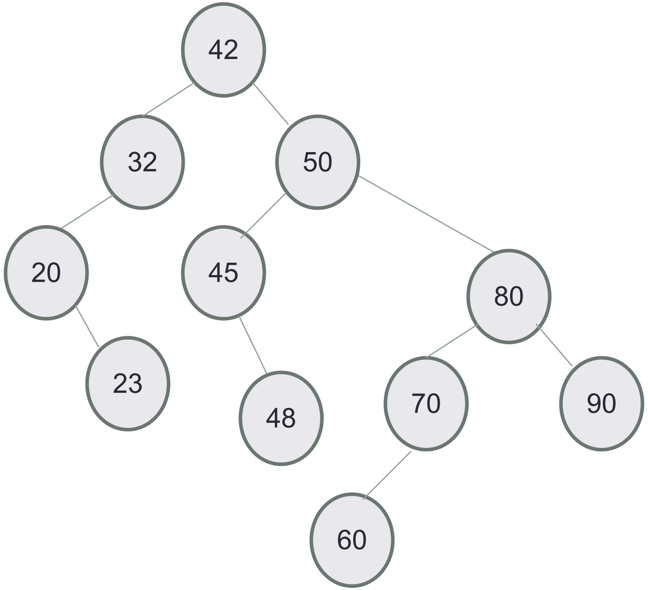
Delete: Case 2



Case 2 Node has only one child

- Replace the node by its only child

Delete: Case 3



Case 3 Node has two children

- Can we still replace the node by one of its children? Why or Why not?