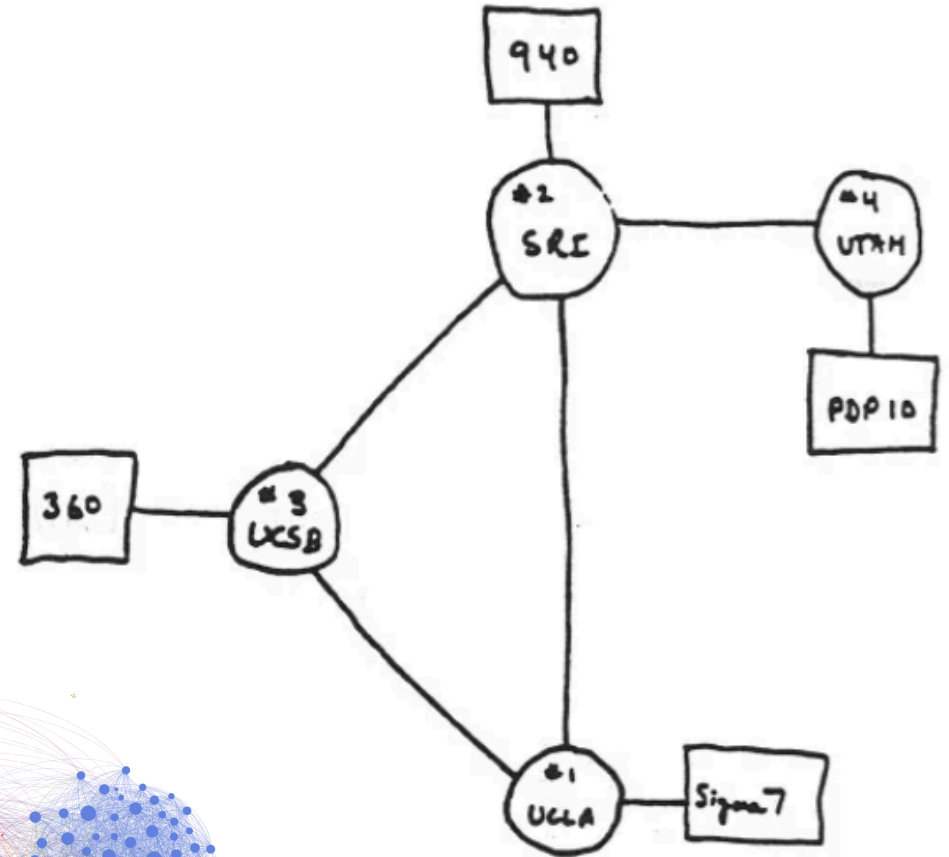
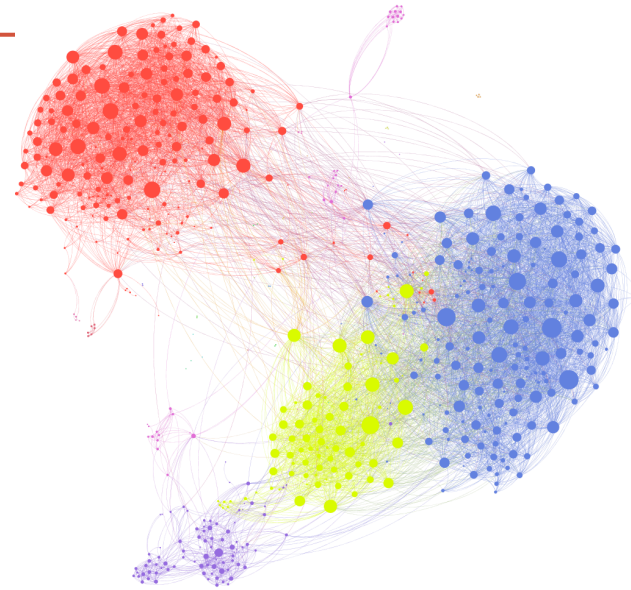
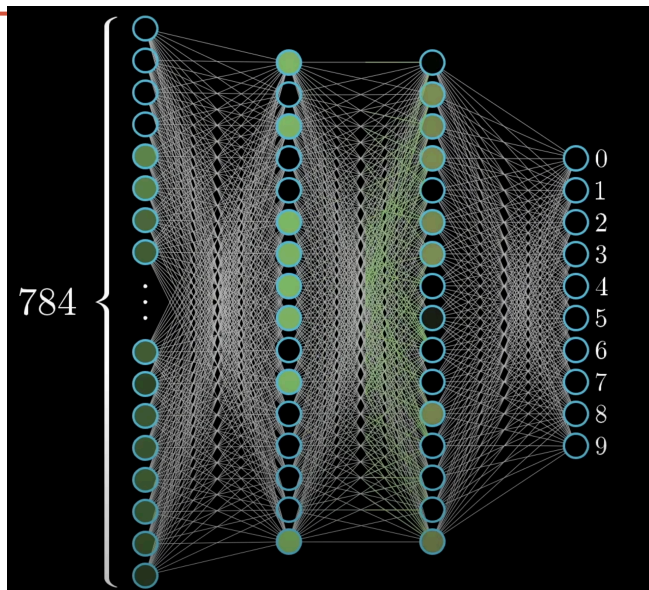


# GRAPHS



THE ARPA NETWORK

DEC 1969

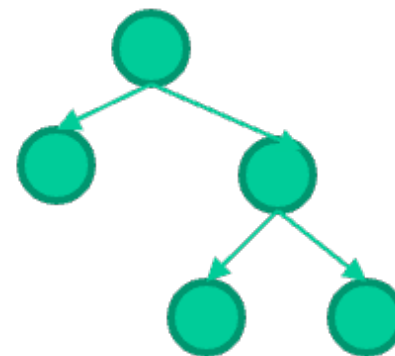
4 NODES

\*The IBM 360, the IMP, and the workstations were all located in North Hall.  
<https://jeweledplatypus.org/news/text/ucsbnet.html>

# Kinds of data structures

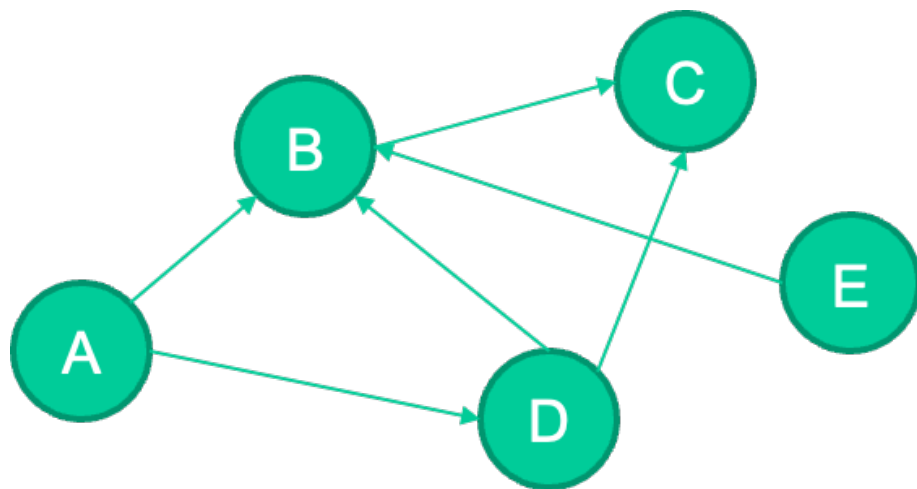


Sequential, linear structures  
(arrays, linked lists)



Hierarchical structures  
(trees)

## Graphs

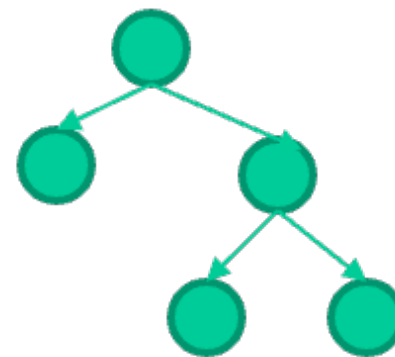


Graphs are not hierarchical or sequential,  
no requirements for a “root” or “parent/child”  
relationships between nodes

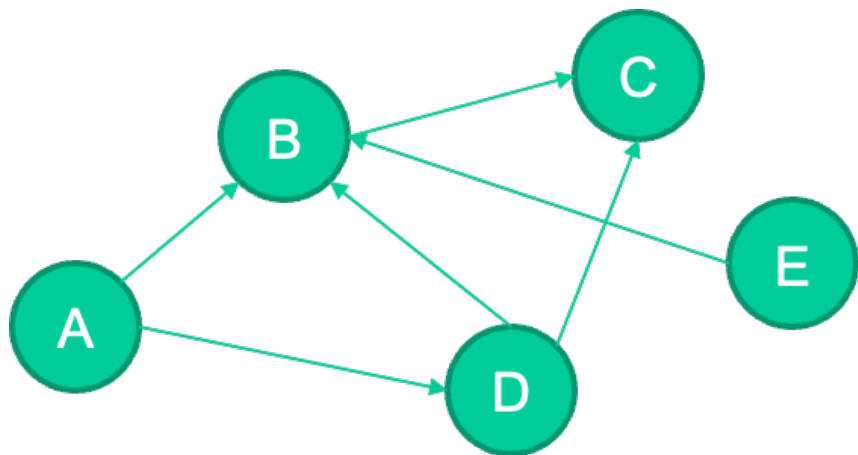
# Kinds of data structures



Sequential, linear structures  
(arrays, linked lists)



Hierarchical structures  
(trees)



## Graphs consist of

- A collection of elements (“nodes” or “vertices”)
- A set of connections (“edges” or “links” or “arcs”) between pairs of vertices.

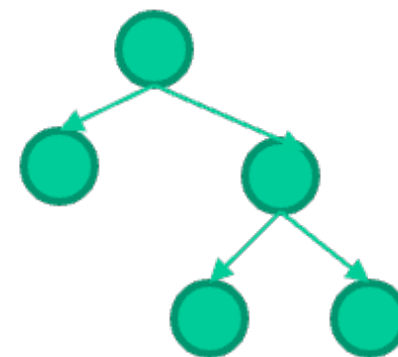
Edges may be directed or undirected

Edges may have weight associated with them

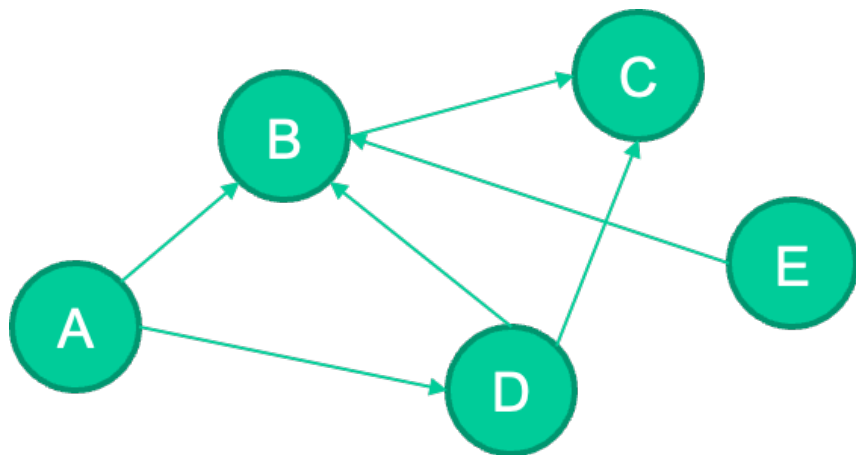
# Kinds of data structures



Sequential, linear structures  
(arrays, linked lists)



Hierarchical structures  
(trees)



## Graphs

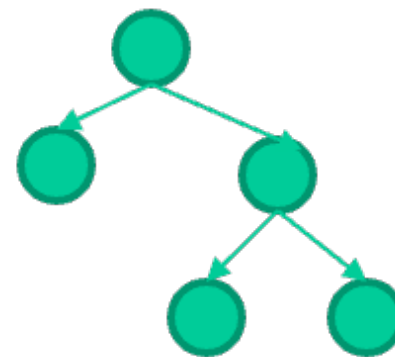
- They consist of both vertices and edges
- They do NOT have an inherent order
- Edges may be weighed or unweighed
- Edges may be directed or undirected
- They may contain cycles



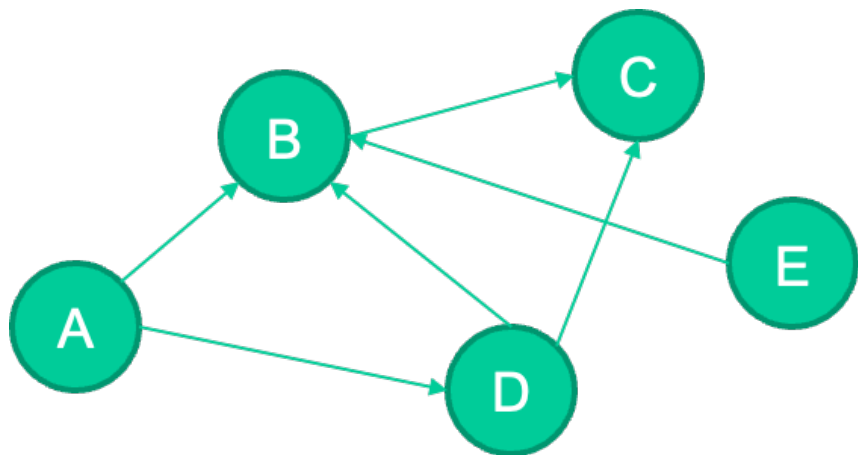
# Kinds of data structures



Sequential, linear structures  
(arrays, linked lists)



Hierarchical structures  
(trees)

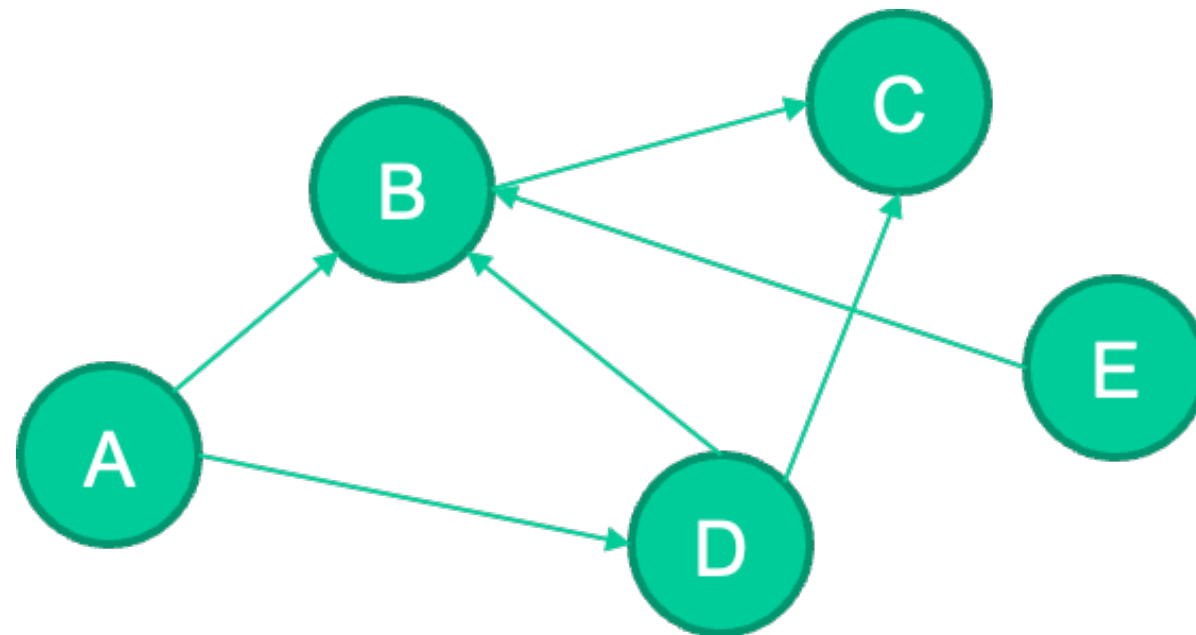


## Graphs

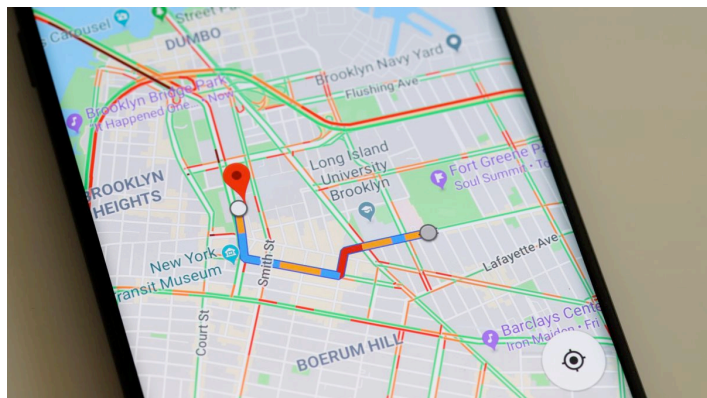
Which of the following is true about graphs?

- A. A graph can always be represented as a tree
- B. A tree can always be represented as a graph
- C. Both A and B
- D. Neither A or B

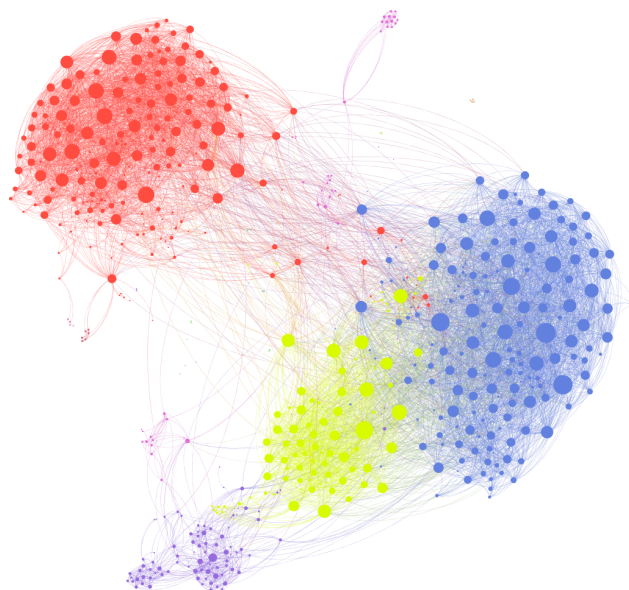
# Why Graphs?



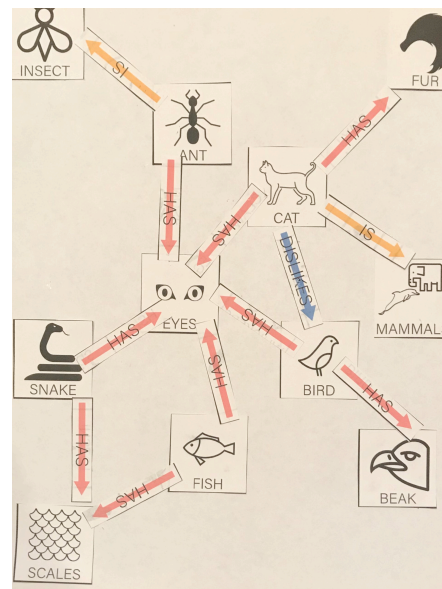
# Why Graphs?



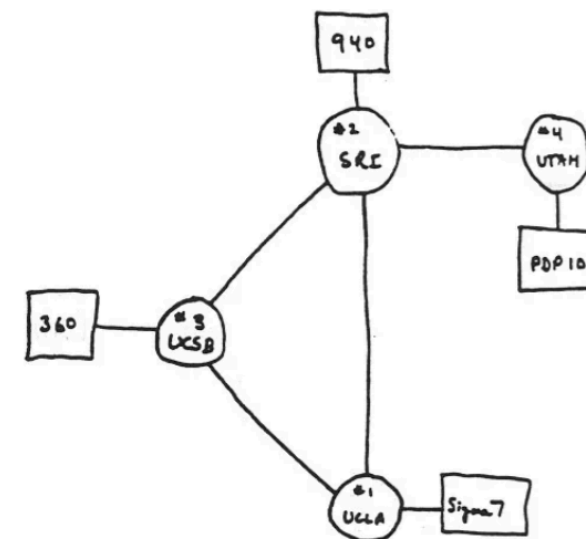
Road networks



Social networks



Semantic networks



THE ARPA NETWORK

DEC 1969

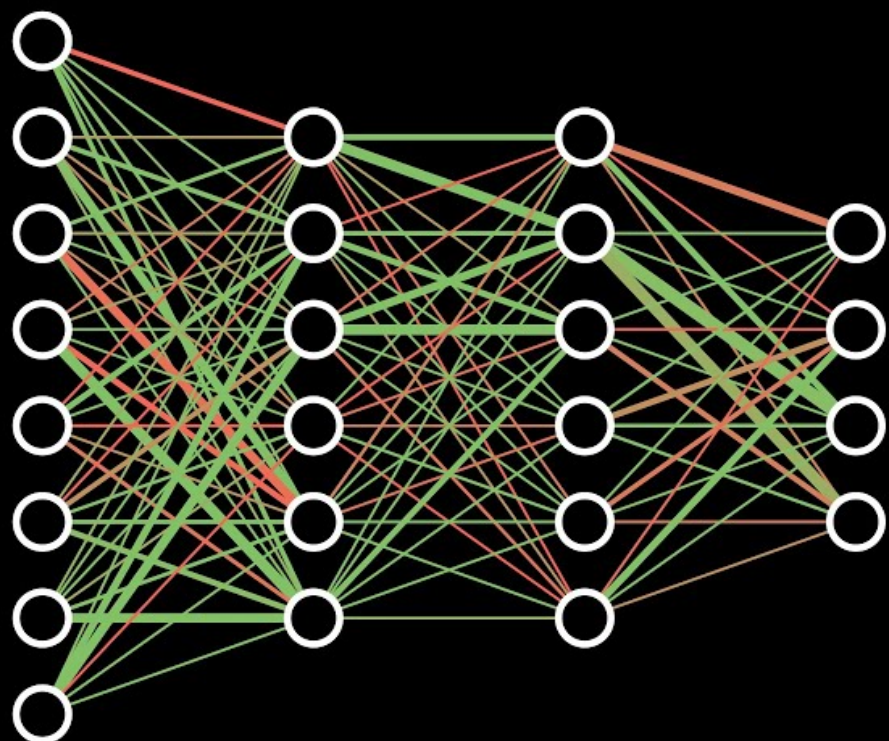
4 NODES

Computer networks\*

Remember: If you can map your problem to a well-known graph problem, it usually means you can solve it fast!

## Next assignment: Graph applications to Machine Learning

# Neural Networks



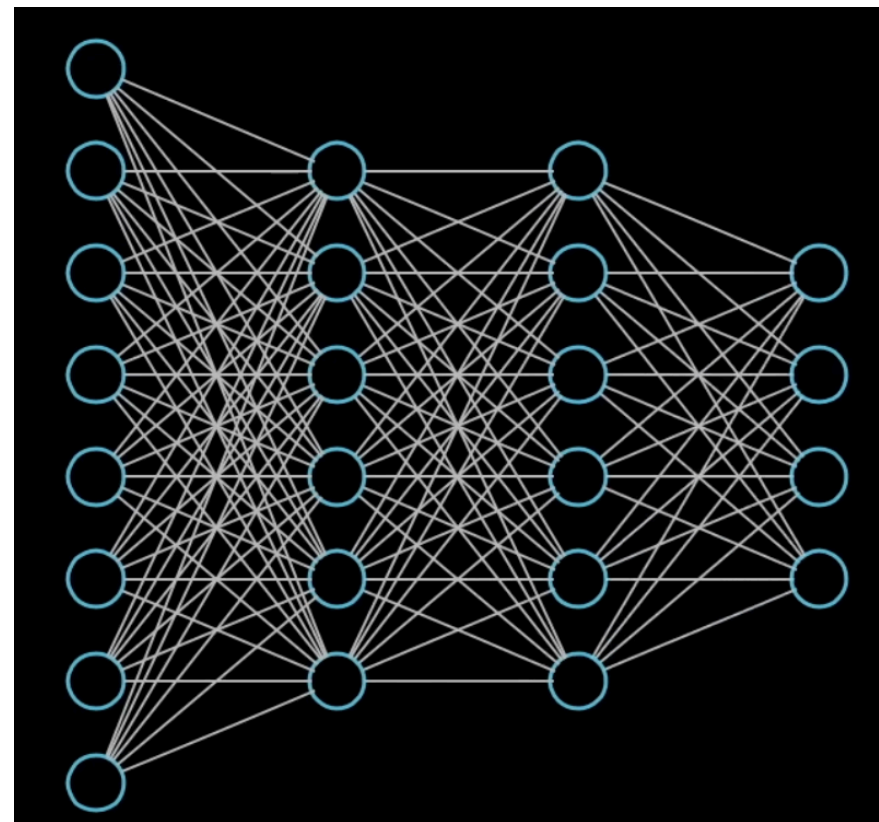
From the  
ground up

<https://youtu.be/aircAruvnKk?feature=shared>

We'll listen to the first 5 minutes

In a graph representing a neural network, which of the following is FALSE? Discuss why in each case.

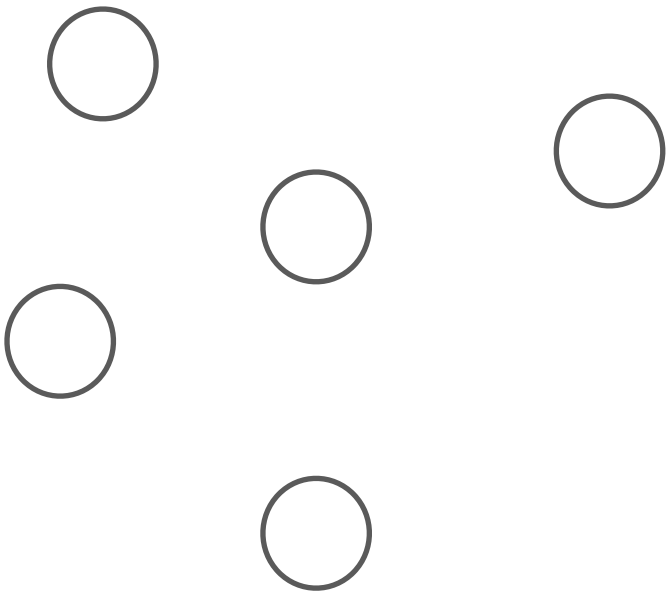
- A. Vertices represent neurons
- B. Edges represent layers
- C. Edges are directed
- D. Edges have weights
- E. None of the above



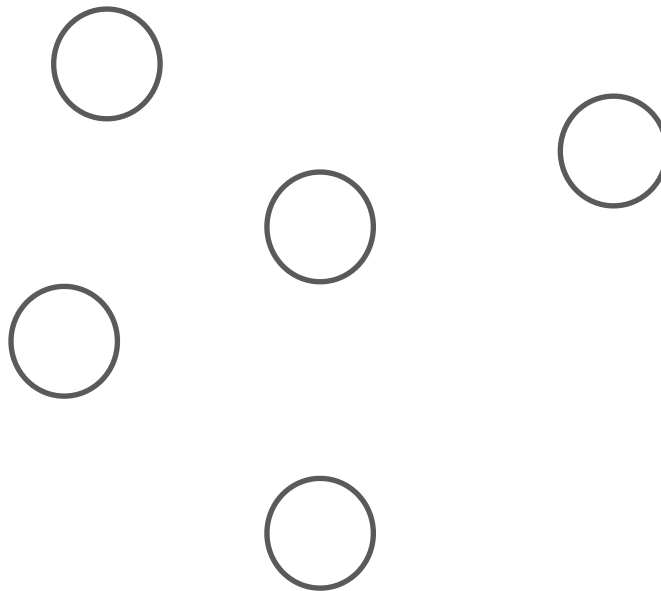
Neural Network

# Types of Graphs

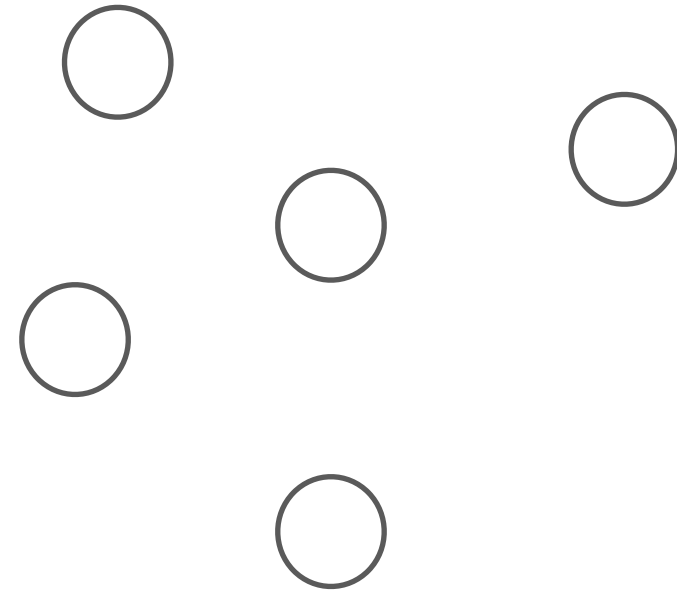
Disconnected



Connected



Fully connected

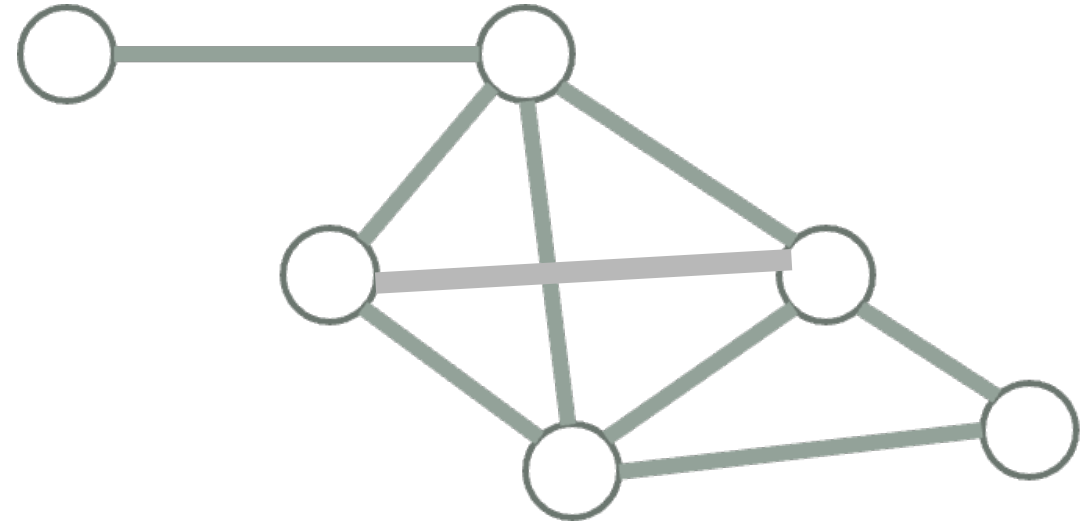
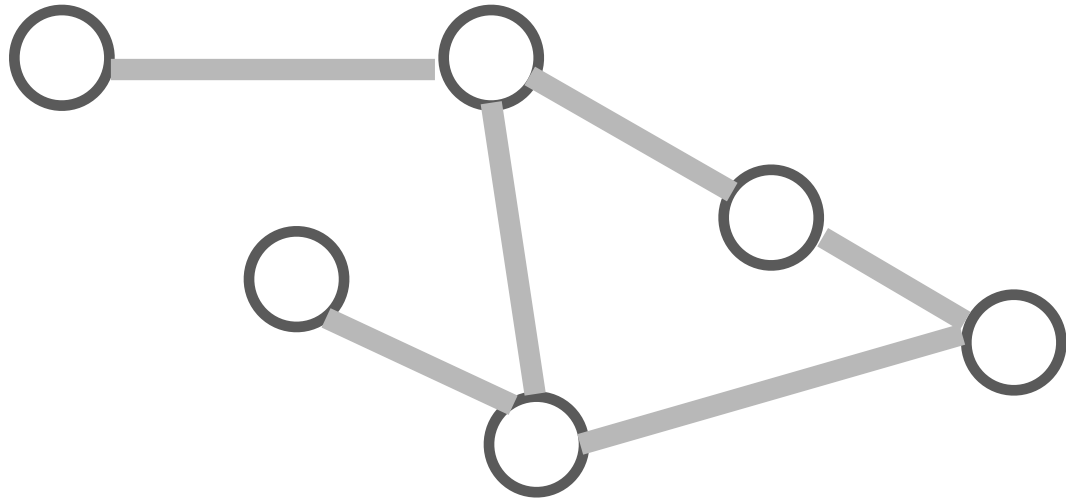


What is minimum and maximum number of edges in a connected undirected graph with  $n$  vertices (with no self-loops)?

- A. 0 and  $n$
- B.  $(n - 1)$  and  $n(n - 1) / 2$
- C.  $(n - 1)$  and  $n^2$
- D.  $(n - 1)$  and  $2^n$



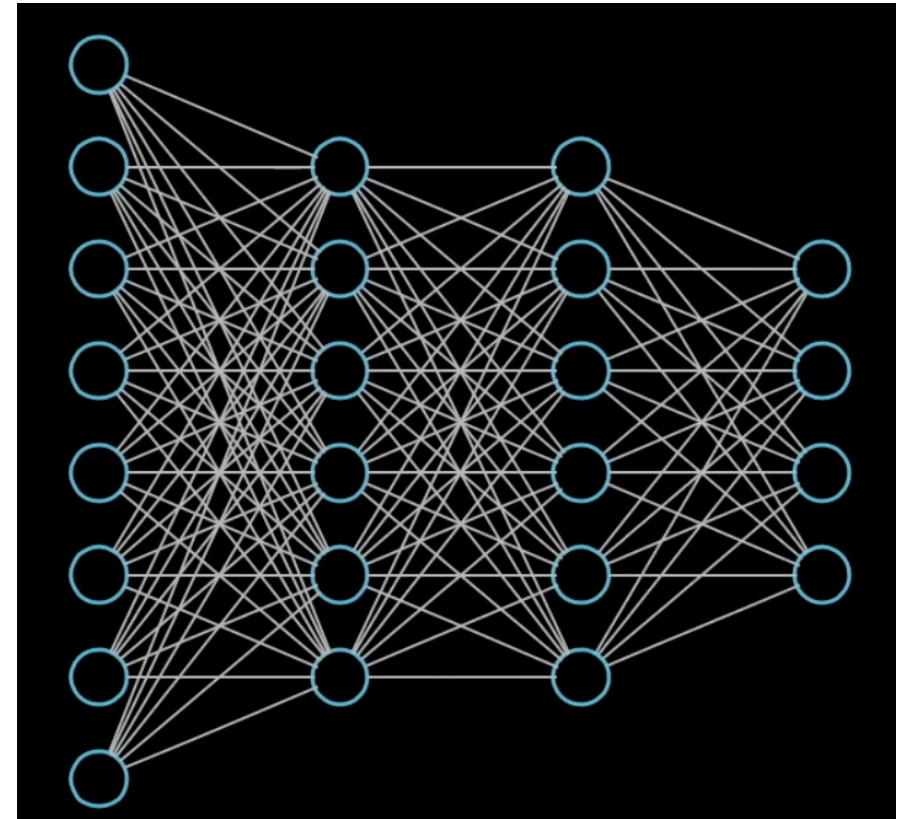
## Sparse vs. Dense Graphs



A dense graph is one where  $|E|$  is “close to”  $|V|^2$ .  
A sparse graph is one where  $|E|$  is “closer to”  $|V|$ .

Is the neural network a sparse or dense graph?

- A. Sparse
- B. Dense
- C. Can't say!



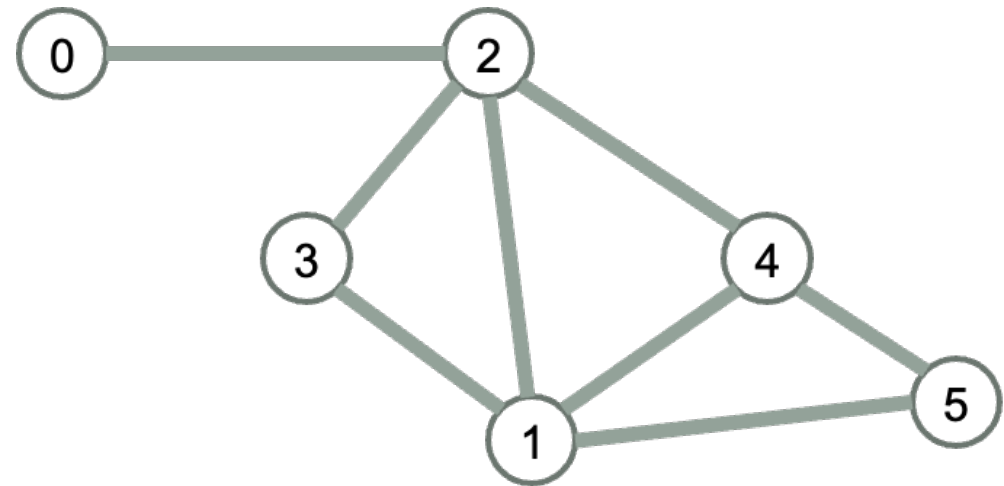
Neural Network

# Adjacency Matrix Representation of a Graph

Represent the graph by a  $n \times n$  binary/integer/float valued adjacency matrix,  $A$

$n$ : number of vertices or  $|V|$

$m$ : number of edges or  $|E|$

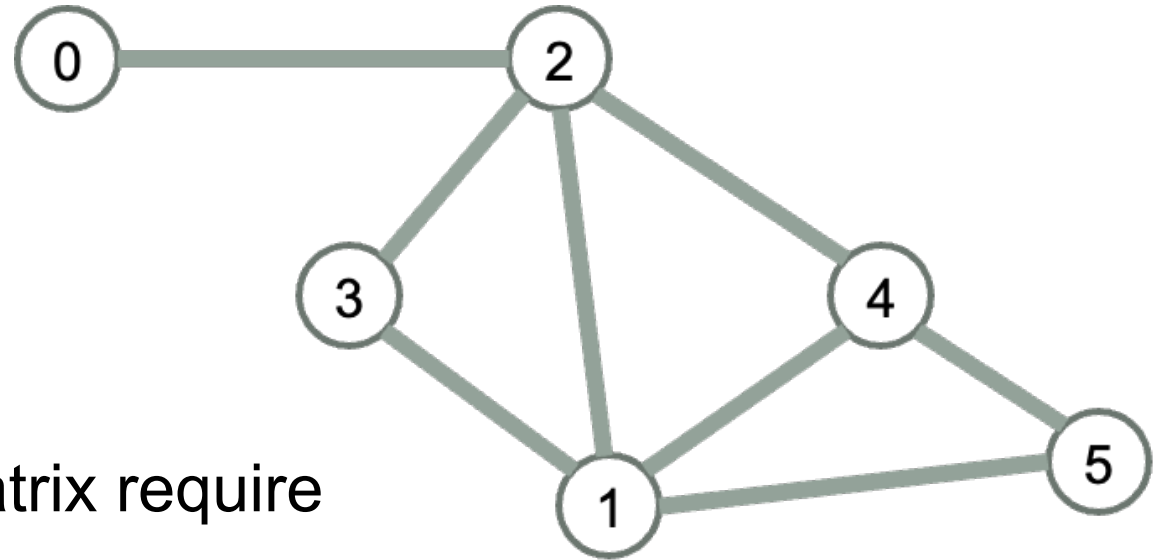



How much space does an adjacency matrix require to represent a graph?

- A.  $O(n)$
- B.  $O(m)$
- C.  $O(n + m)$
- D.  $O(n^2)$
- E.  $O(mn)$

# Adjacency Matrix

Represent the graph by a  $n \times n$  binary valued adjacency matrix,  $A$   
 $A[i, j] = 1$ , if there is an edge from  $i$  to  $j$



How much space does an adjacency matrix require to represent a graph?

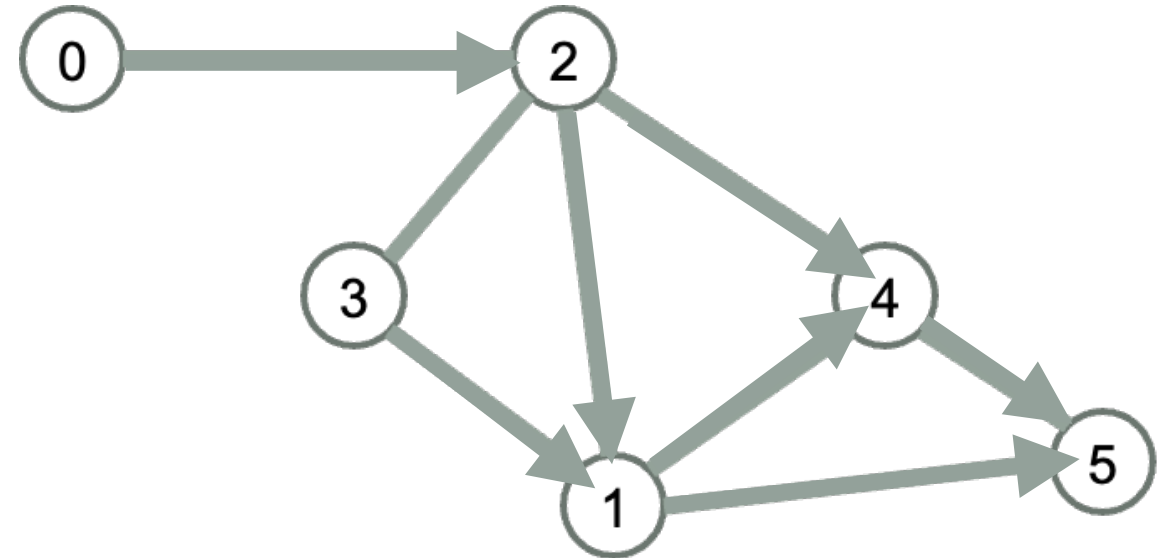
- A.  $O(n)$
- B.  $O(m)$
- C.  $O(n + m)$
- D.  $O(n^2)$
- E.  $O(mn)$

$n$ : number of vertices or  $|V|$   
 $m$ : number of edges or  $|E|$

# Adjacency List Representation of a Graph

- Vertices and edges stored as lists
- Each vertex points to all its edges

0	
1	
2	
3	
4	
5	

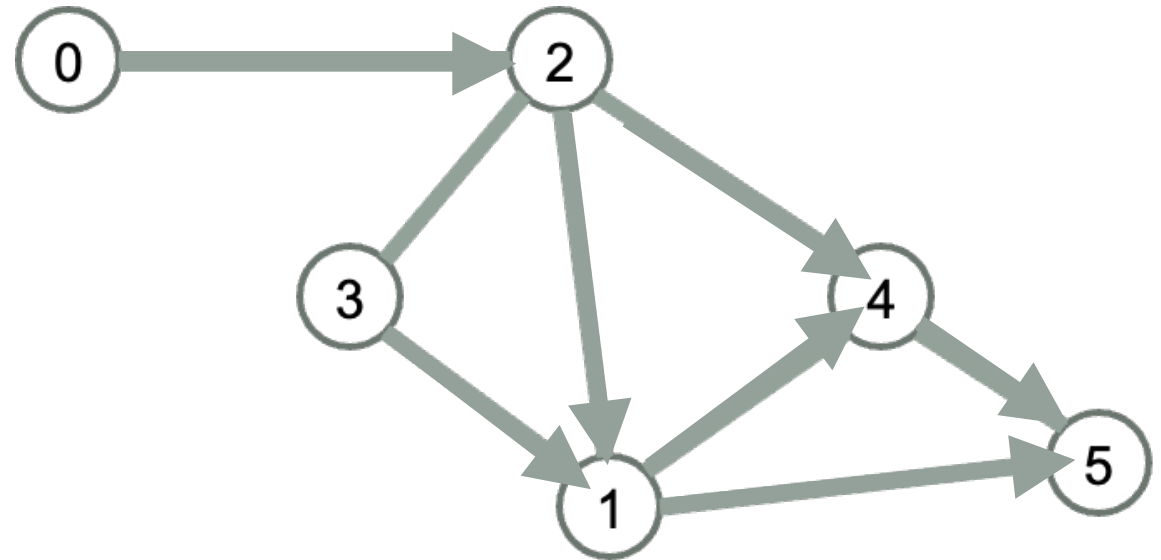


How much space does an adjacency list require to represent a graph?

- A.  $O(n)$
- B.  $O(m)$
- C.  $O(n + m)$
- D.  $O(n^2)$
- E.  $O(m.n)$

Assume each vertex has a unique id between 0 and 5

```
class graph{  
    private:  
    _____ adjlist;  
};
```



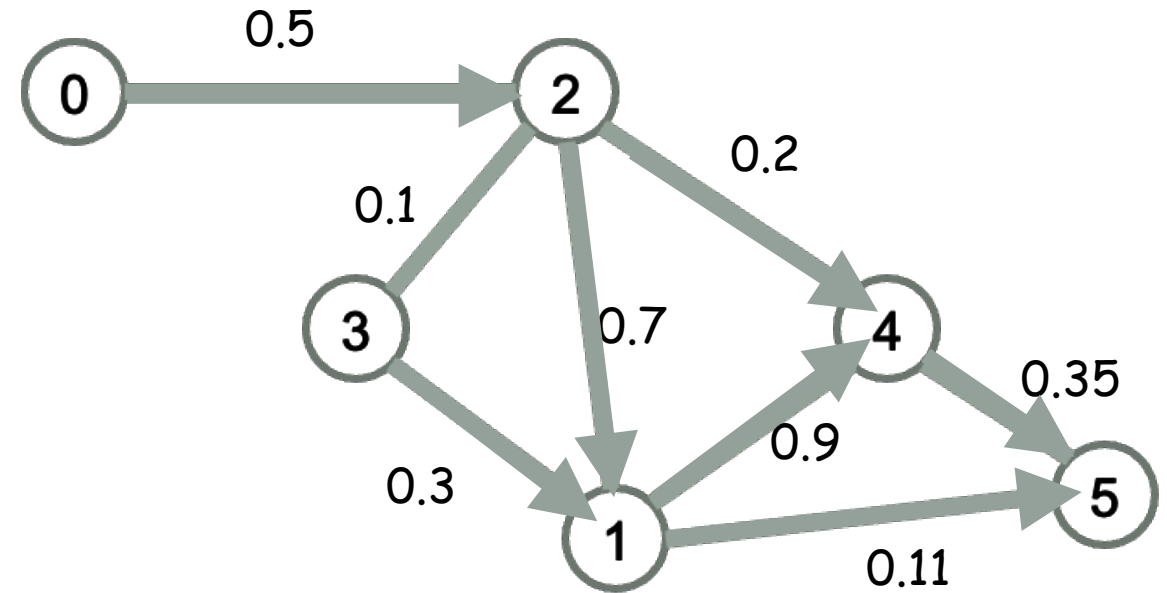
Choose the ADT to represent the adjacency list

- A. `vector<int>`
- B. `vector<unordered_set<int>>`
- C. `list<vector<int>>`
- D. `vector<list<int>>`
- E. `set<list<int>>`

# Adjacency List: Weighted graph

- Vertices and edges stored as lists
- Each vertex points to all its edges

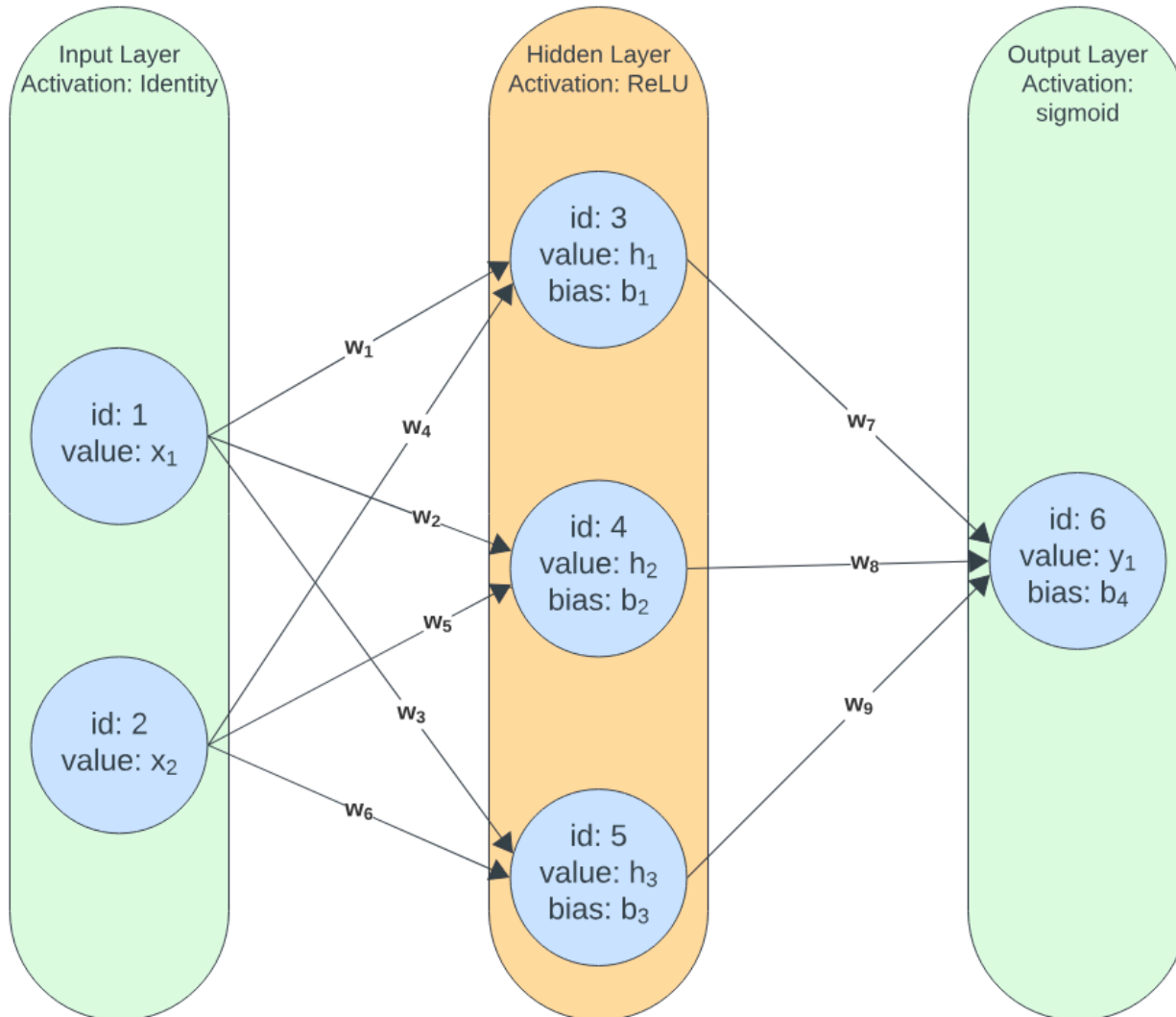
0	
1	
2	
3	
4	
5	





# Neural Network structure for upcoming assignment

```
typedef std::vector<std::unordered_map<int, Connection> > AdjList;
```



# Understanding the Graph and NeuralNetwork classes

```
typedef std::vector<std::unordered_map<int, Connection> > AdjList;
```

```
class Graph {
    public:
        Graph();
        Graph(int size);
        // Constructors and destructor

        // TODO: graph methods
        void updateNode(int id, NodeInfo n);
        NodeInfo* getNode(int id) const;
        void updateConnection(int v, int u, double w);

    protected:
        // protected to give NeuralNetwork access

        // adjacency list containing weights for edges.
        AdjList adjacencyList;

        // vector storing node info
        std::vector<NodeInfo*> nodes;

        //Other functions
};
```

```
class NeuralNetwork : public Graph {
    public:
        // Constructors and public functions

    private:
        // each index of layers holds a vector which
        // contains the id's of every node in that layer.
        std::vector<std::vector<int> > layers;

        // contains ids of input nodes
        std::vector<int> inputNodeIds;

        // contains ids of output nodes
        std::vector<int> outputNodeIds;

        // since NeuralNetwork inherits from Graph, you can imagine
        // all of the graph members here as well...

};
```

## Activity: Draw the final NN by hand

```
void test_algorithm() {
    cout << "test_algorithm" << endl;
    NeuralNetwork nn(6);

    NodeInfo n0("ReLU", 0, -0.2);
    NodeInfo n1("ReLU", 0, 0.2);
    NodeInfo n2("identity", 0, 0);
    NodeInfo n3("sigmoid", 0, 0.98);
    NodeInfo n4("ReLU", 0, 0.11);
    NodeInfo n5("identity", 0, 0);

    nn.updateNode(0, n0);
    nn.updateNode(1, n1);
    nn.updateNode(2, n2);
    nn.updateNode(3, n3);
    nn.updateNode(4, n4);
    nn.updateNode(5, n5);

    nn.updateConnection(2, 1, 0.1);
    nn.updateConnection(2, 4, 0.2);
    nn.updateConnection(2, 0, 0.3);
    nn.updateConnection(5, 1, 0.4);
    nn.updateConnection(5, 4, 0.5);
    nn.updateConnection(5, 0, 0.6);
    nn.updateConnection(1, 3, 0.7);
    nn.updateConnection(4, 3, 0.8);
    nn.updateConnection(0, 3, 0.9);

    nn.setInputNodeIds({2, 5});
    nn.setOutputNodeIds({3});
}
```

Next, map out the information stored in:

- `nn.nodes`
- `nn.adjacencyList`
- `nn.inputNodeIds`
- `nn.outputNodeIds`