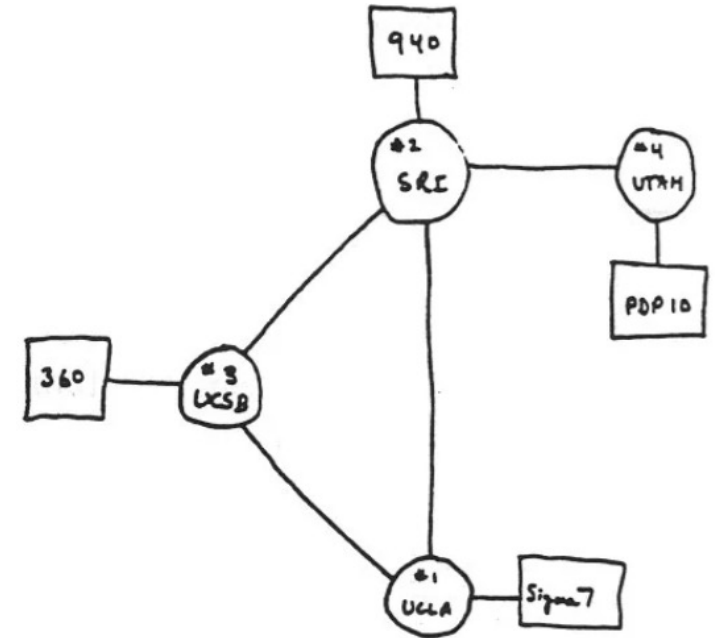
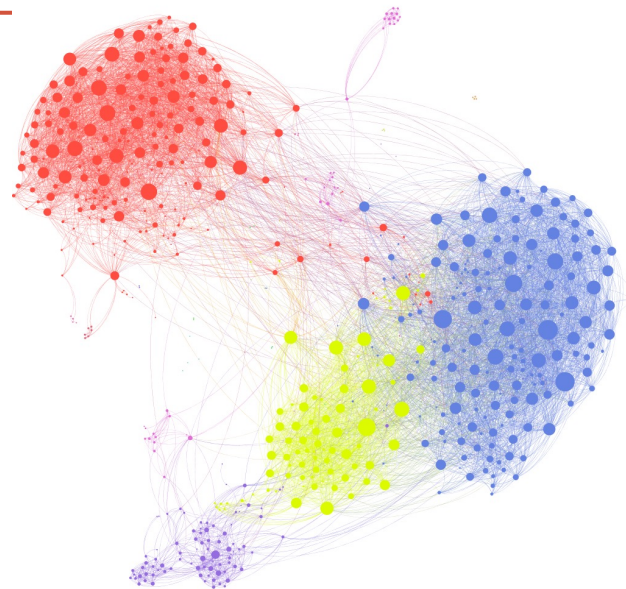
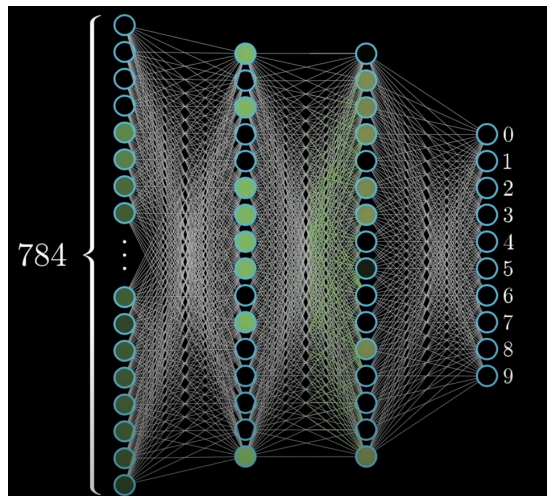


MERGE SORT



THE ARPA NETWORK

DEC 1969

4 NODES

Divide and Conquer Algorithms

Algorithm Approach:

- **Divide** a large problem into sub-problems
- **Solve** each sub-problem
- **Combine** the solutions of sub-problems to obtain the solution for the original problem

Merge Sort Algorithm

`MergeSort(vector v)`

- Divide `v` into left half and right half
- Sort the left half, then sort the right half
- Combine (merge) the two sorted halves

[7 2]

Example run of
mergesort

[7 2 3 -1]

Example run of
mergesort

[7 2 5 3 -1]

Example run of mergesort

What is the maximum depth of the binary tree trace of mergeSort?

Generalize the answer for an input vector of size n

- A. 1. B. 2 C. 3 D. 4 E. 5

Running Time Analysis

[7 2 5 3 -1]

$T(n) =$ # operations to split each list +
comparisons to merge lists +
function calls

[7 2 5] [3 -1]

[7 2] [5] [3] [-1]

[7] [2]

Space Analysis

[7 2 5 3 -1] $S(n) =$

[7 2 5] [3 -1]

[7 2] [5] [3] [-1]

[7] [2]

How much additional space is used by the time mergesort reaches the base case?

- A. $n \cdot \log(n)$
- B. $n + n/2 + n/4 + n/8 + \dots + 1$
- C. $n + n/2 + n/4 + n/8 + \dots + 1 + \log(n)$
- D. Something else

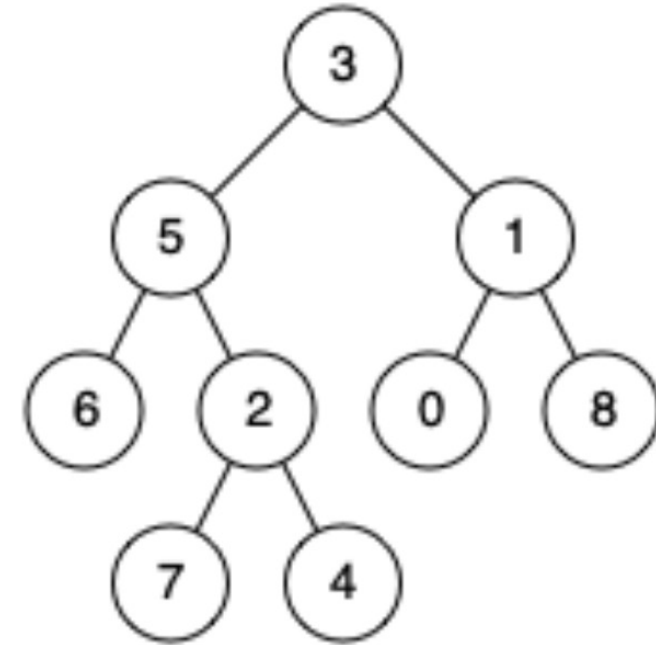
Path: a sequence of nodes in which each node is connected by an edge to the next.

Ancestor(u): any node that is on a path ending in u

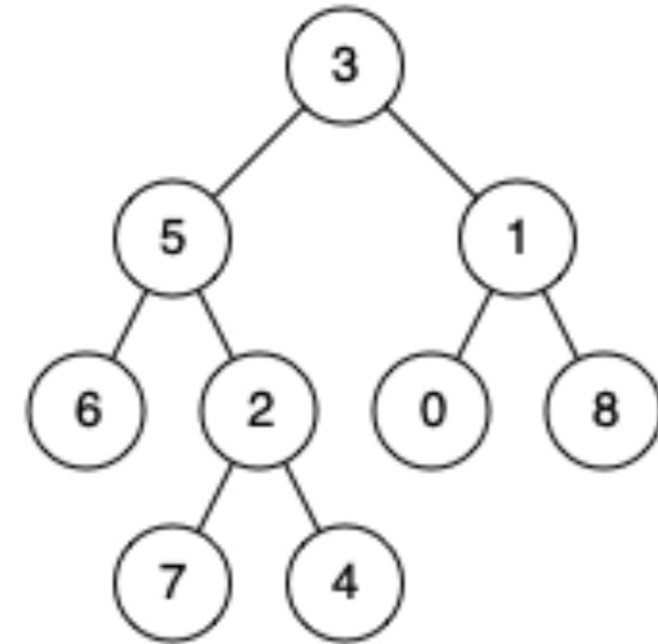
Descendant(v): any node that is on a path starting from v

Common ancestor(u, v): any node that is the ancestor of both u and v

Lowest Common ancestor(u, v): deepest node in the tree that is a common ancestor of u and v



Approach 1: Turn definitions into an algorithm



Approach 2: Divide and Conquer

