SPACE COMPLEXITY
BEST & WORST CASE ANALYSIS

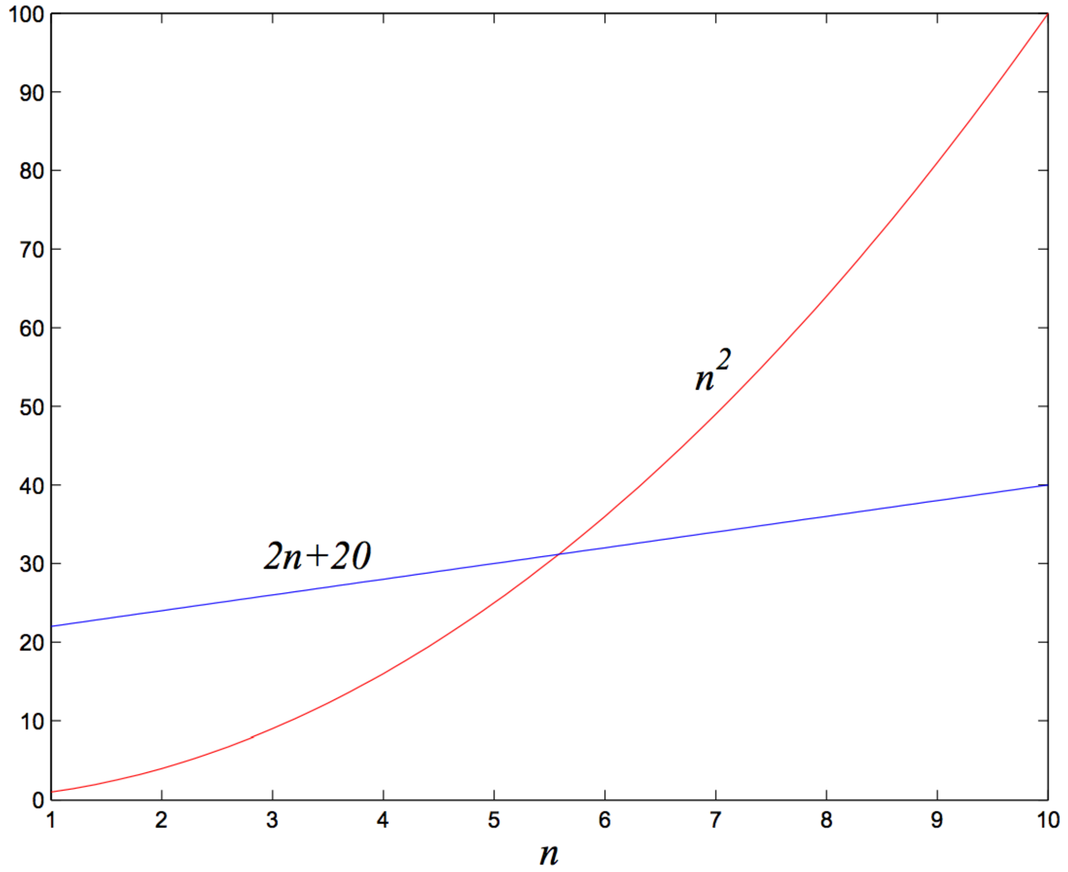
Problem Solving with Computers-II

Definition of Big-O

f(n) and g(n) map positive integer inputs to positive reals.

We say $f = O(g)$ if there is a constant $c > 0$ and $k>0$ such that $f(n) \leq c \cdot g(n)$ for all $n >= k$.

$f = O(g)$
means that "f grows no faster than g"

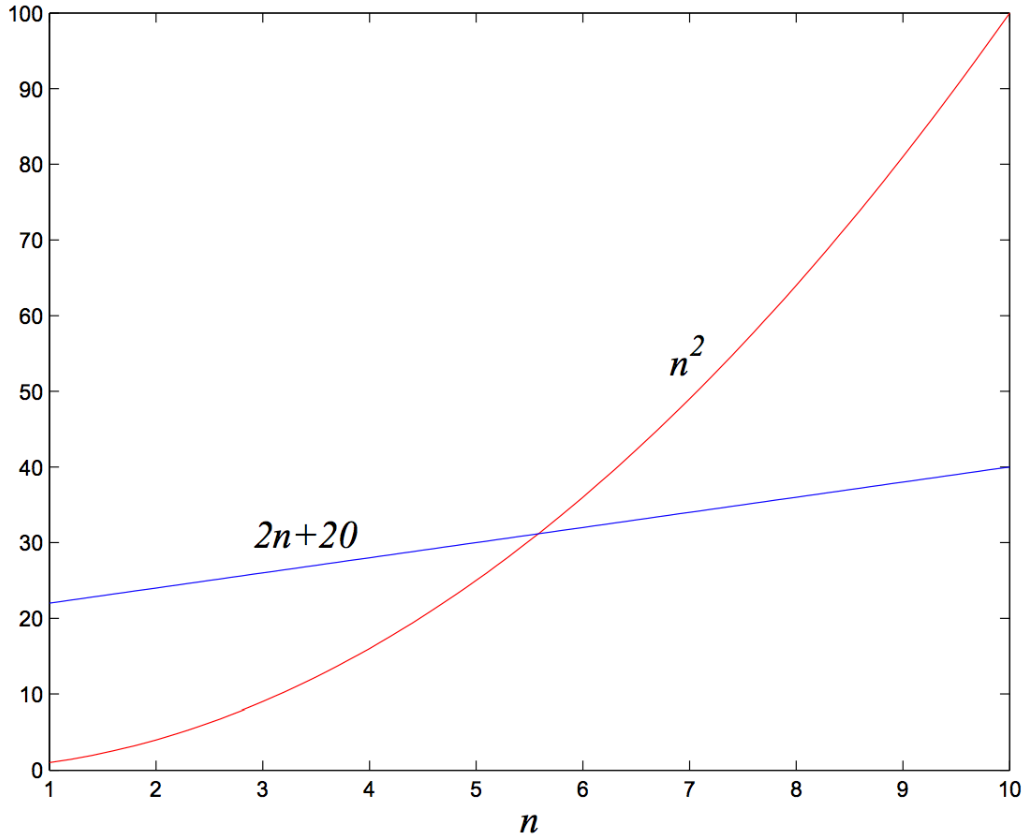


Big-Omega

- f(n) and g(n) map positive integer inputs to positive reals.

We say $f = \Omega(g)$ if there are constants $c > 0$, $k > 0$ such that
$c \cdot g(n) \leq f(n)$ for $n >= k$

$f = \Omega(g)$
means that "f grows at least as fast as g"

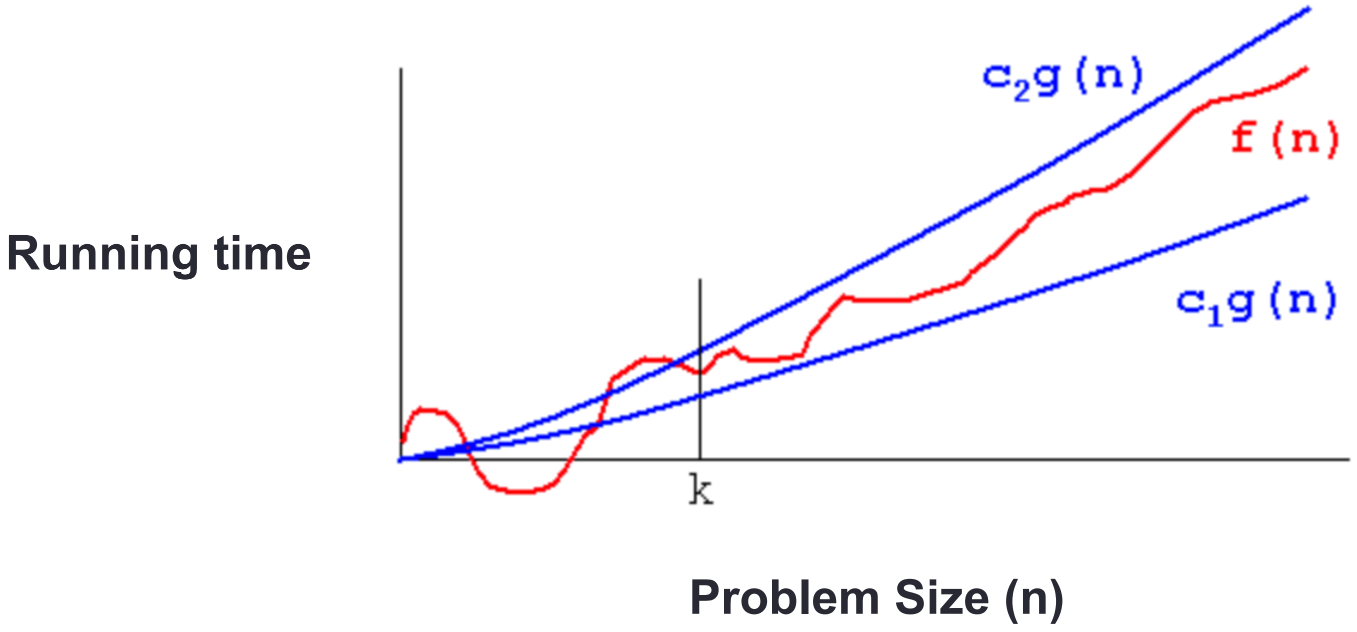


Big-Theta

- f(n) and g(n) map positive integer inputs to positive reals.

We say $f = \Theta(g)$ if there are constants c_1, c_2, k such that

$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$, for n >=k



Space Complexity

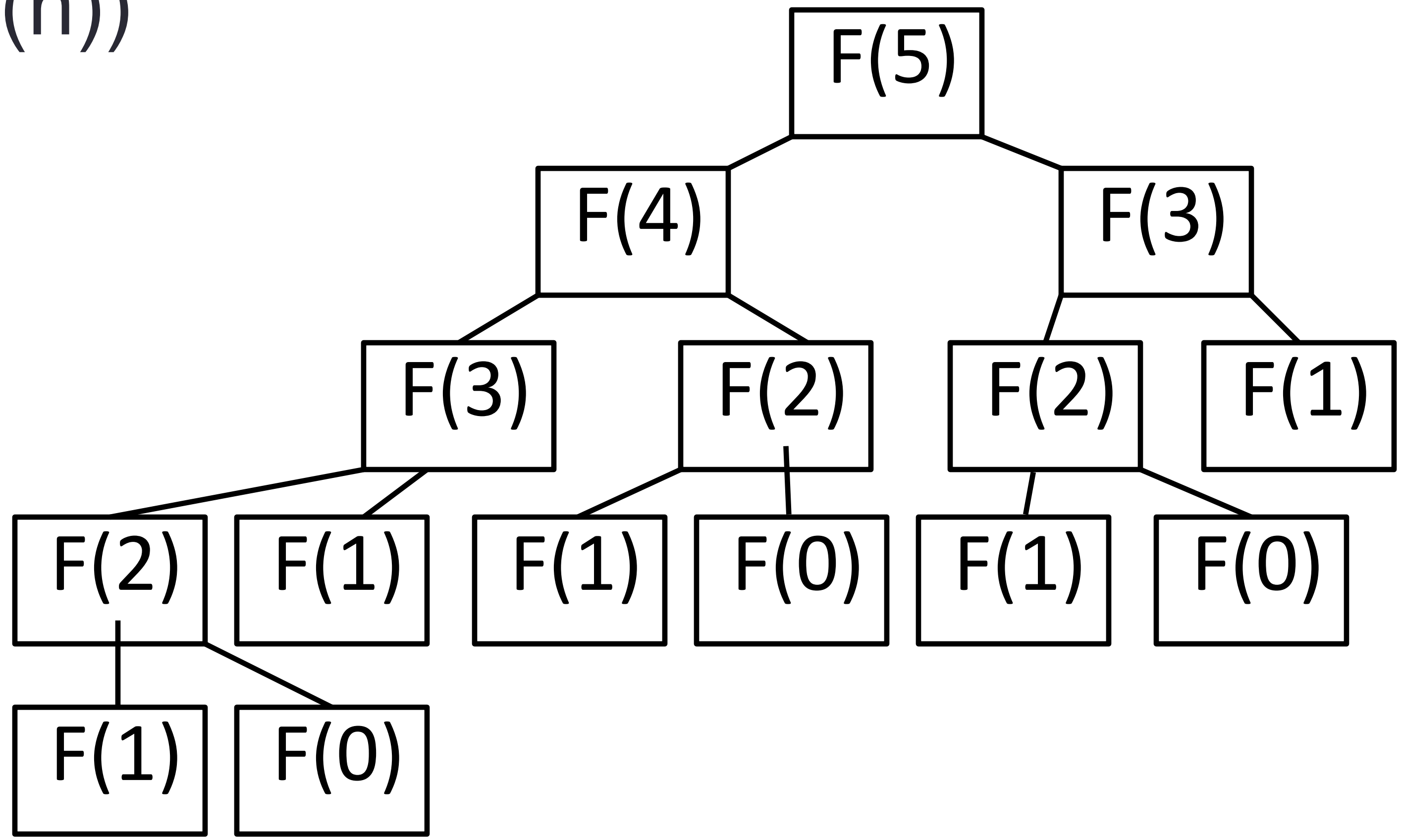
Lets S(n) = maximum amount of memory needed to compute F(.)

```
F(int n){
   if(n <= 1) return 1
   return F(n-1) + F(n-2)
}
```

What is S(n)? Express your answer in Big-O notation

What is S(n)? Express your answer in Big-O notation

A. O(1)
B. O(log(n))
C. O(n)
D. O(n^2)
E. O(2^n)



Tree of recursive calls needed to compute F(5)

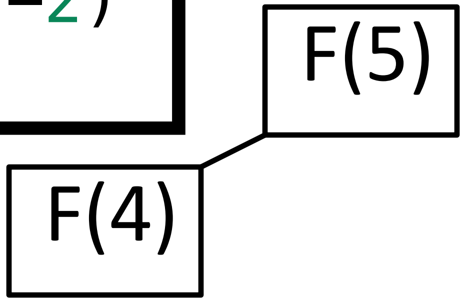
S(n) relates to maximum depth of the recursion

```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```

F(5)

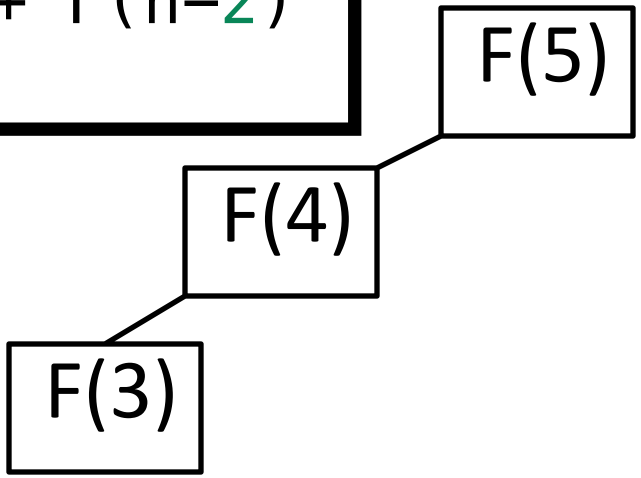
S(n) relates to maximum depth of the recursion

```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



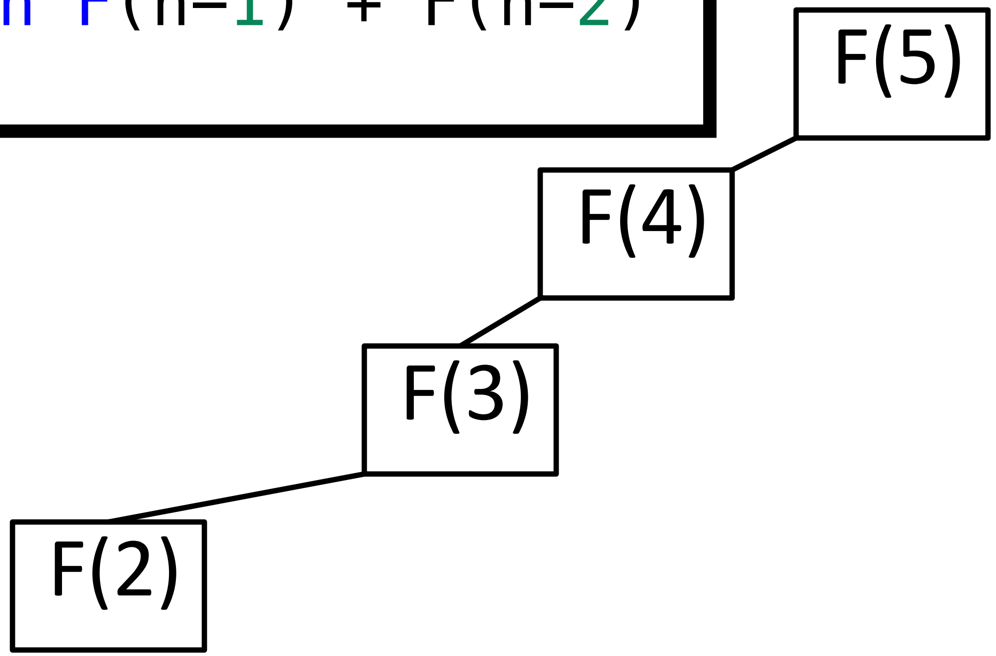
S(n) relates to maximum depth of the recursion

```
F(int n){
   if(n <= 1) return 1
   return F(n-1) + F(n-2)
}
```



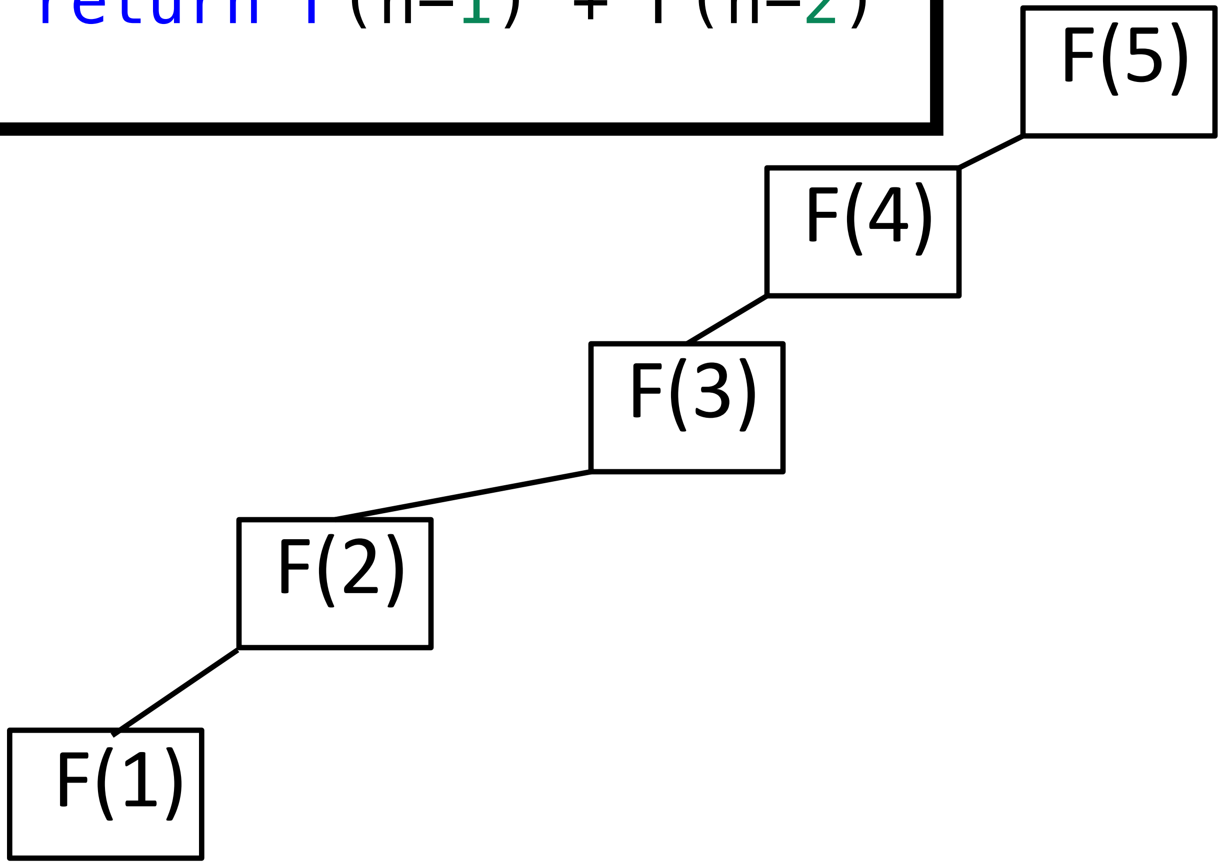
S(n) relates to maximum depth of the recursion

```
F(int n){
   if(n <= 1) return 1
   return F(n-1) + F(n-2)
}
```



S(n) relates to maximum depth of the recursion

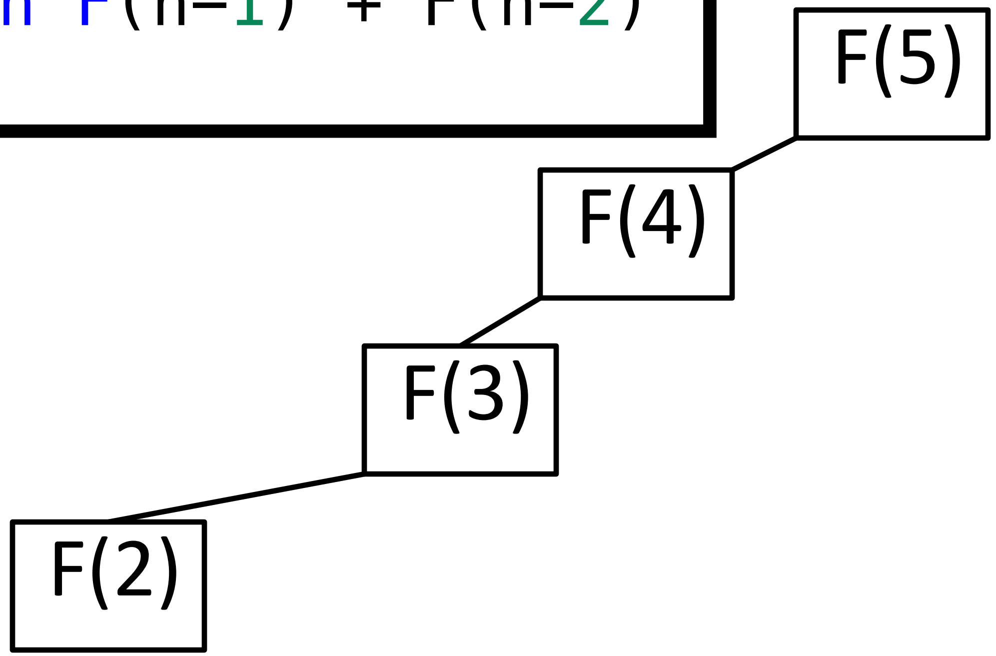
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

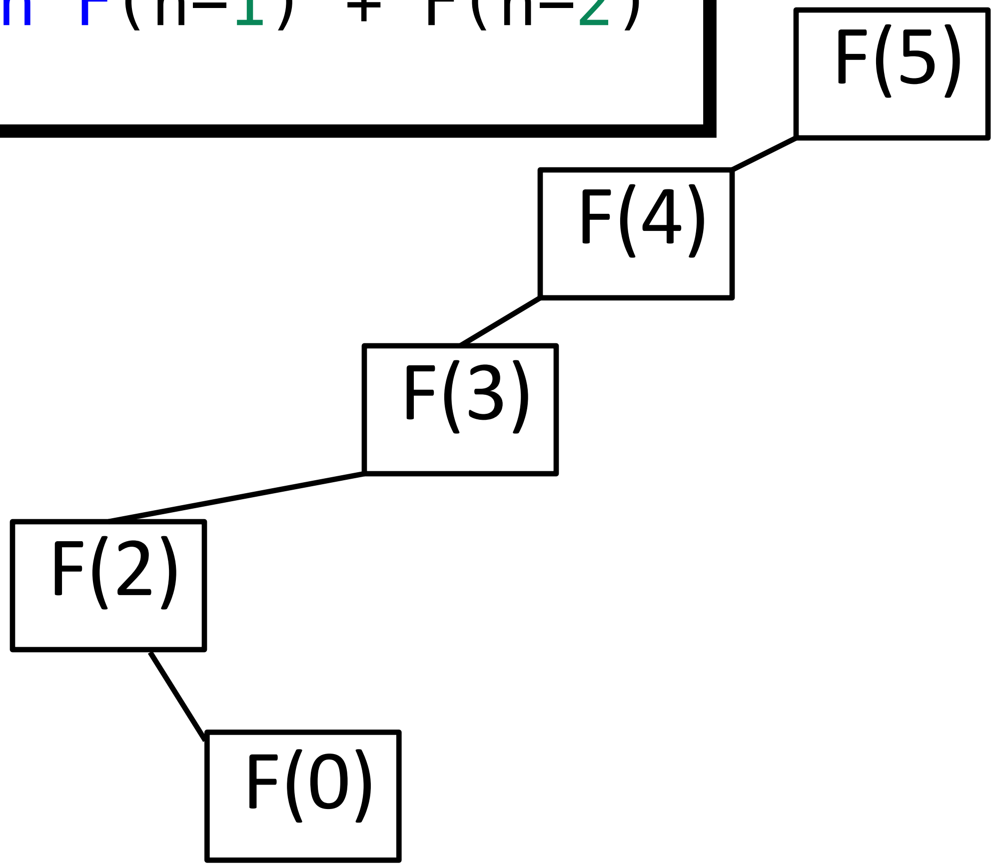
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

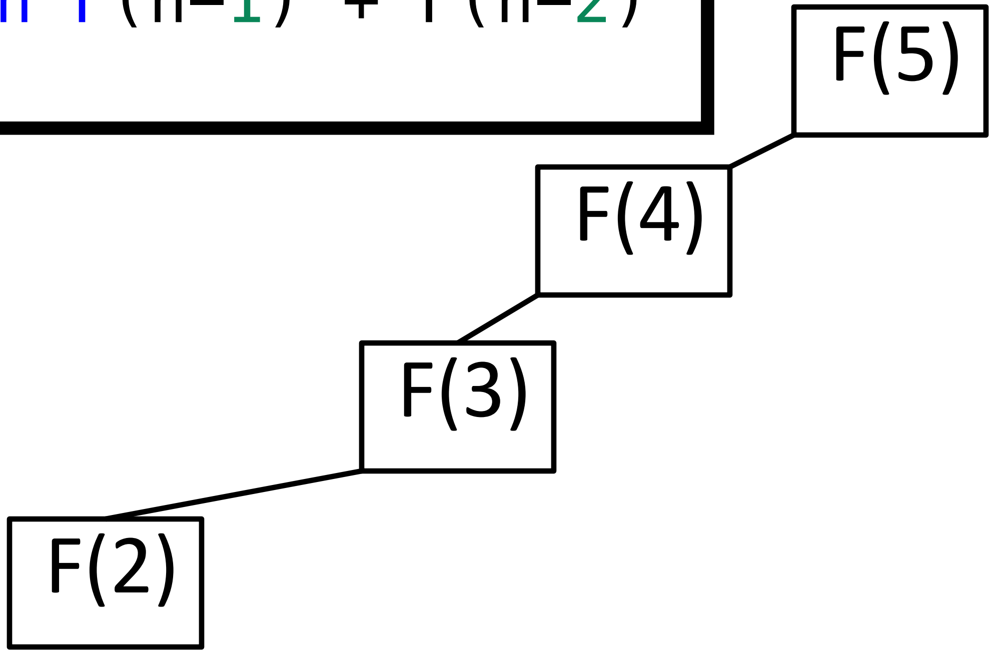
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

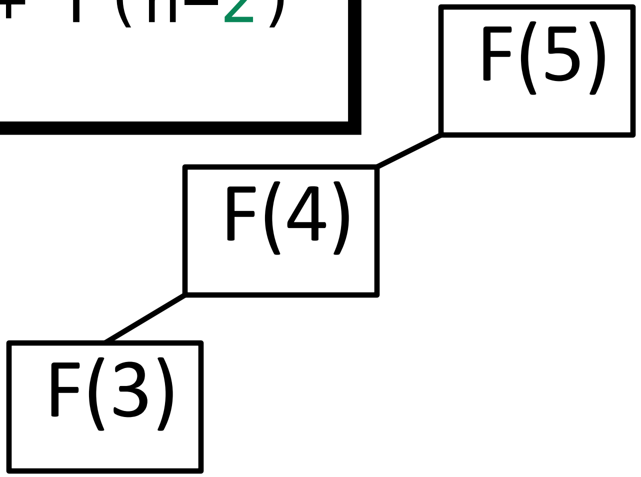
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

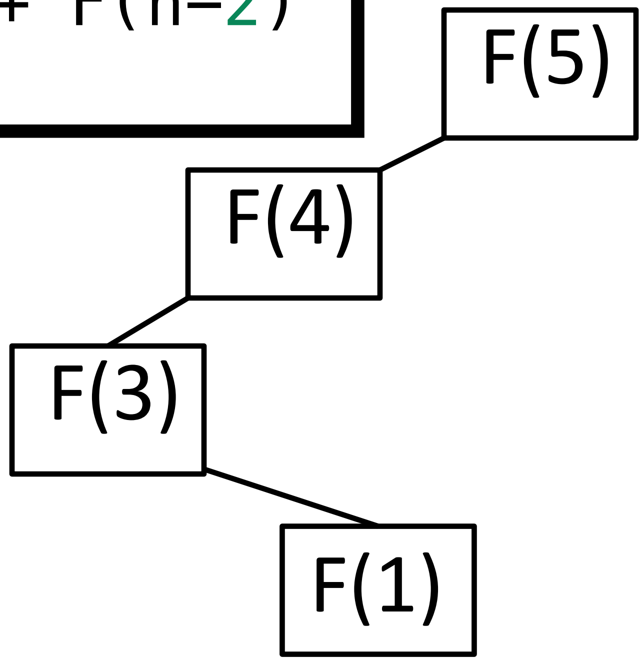
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

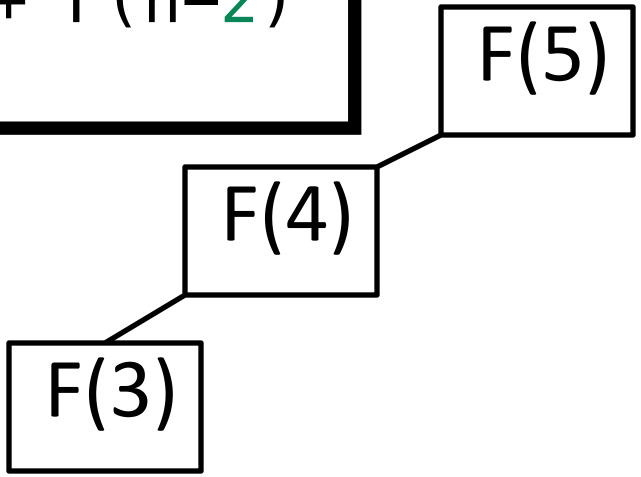
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

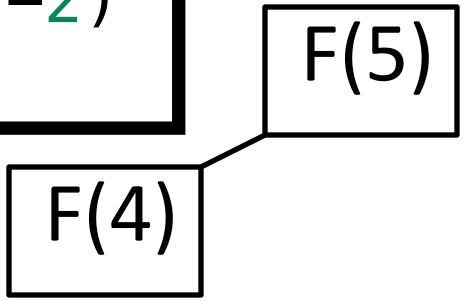
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

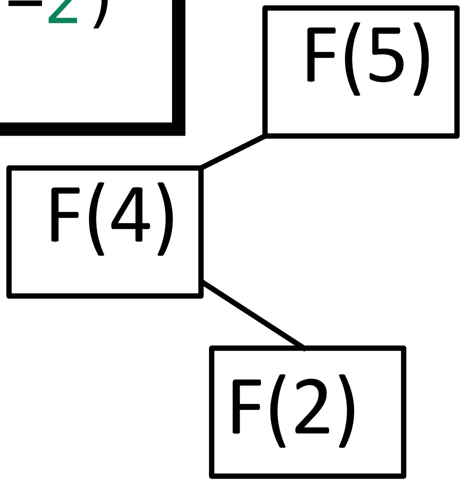
```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

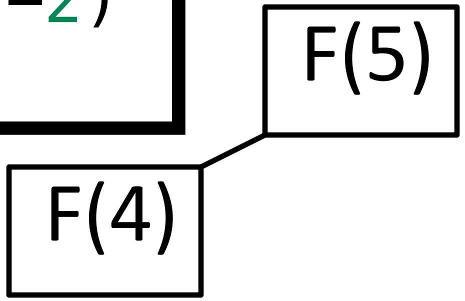
```
F(int n){
   if(n <= 1) return 1
   return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Maximum depth of the recursion = 5

S(n) relates to maximum depth of the recursion

```
F(int n){
   if(n <= 1) return 1
   return F(n-1) + F(n-2)
}
```

F(5)

What is the next step ?

A. Recursion ends and F(5) returns

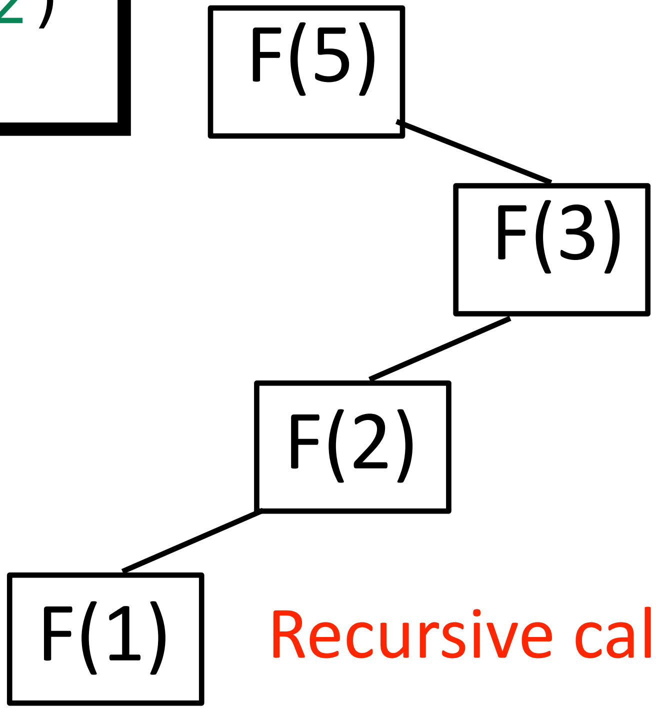
B. F(5) calls F(4)

C. F(5) calls F(3)

D. None of the above

S(n) relates to maximum depth of the recursion

```
F(int n){
    if(n <= 1) return 1
    return F(n-1) + F(n-2)
}
```



Recursive calls continue

Maximum depth of the recursion for F(n) = n

Therefore, S(n) = O(n)

What is the Big-O running time of search in a sorted array of size n?

…using linear search?

…using binary search?

6	13	14	25	33	43	51	53	64	72	84	93	95	96	97
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Worst case analysis of binary search

```
bool binarySearch(int arr[], int element, int n){
//Precondition: input array arr is sorted (ascending order)
  int begin = 0;
  int end = n-1;
  int mid;
  while (begin <=  end){
    mid = (end + begin)/2;
    if(arr[mid]==element){
      return true;
    }else if (arr[mid]< element){
      begin = mid + 1;
    }else{
      end = mid - 1;

    }
  }
  return false;
}
```

Running time of operations in a sorted array

	Best case	Worst case
Search (Binary search)		
Min/Max		
Median		
Successor/Predecessor		
Insert		
Delete		

6	13	14	25	33	43	51	53	64	72	84	93	95	96	97
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

```
procedure max(a1,a2, … an: integers)
 max:= a1
 for i:= 2 to n
     if max < ai
         max:= x
return max{max is the greatest element}
```

What is the **best case** Big-O running time of max?

A. $O(1)$
B. $O(\log n)$
C. $O(n)$
D. $O(n^2)$
E. None of the above

```
procedure max(a1,a2, … an: integers)
 max:= a1
 for i:= 2 to n
     if max < ai
        max:= x
return max{max is the greatest element}
```

What is the **worst case** Big-O running time of max?

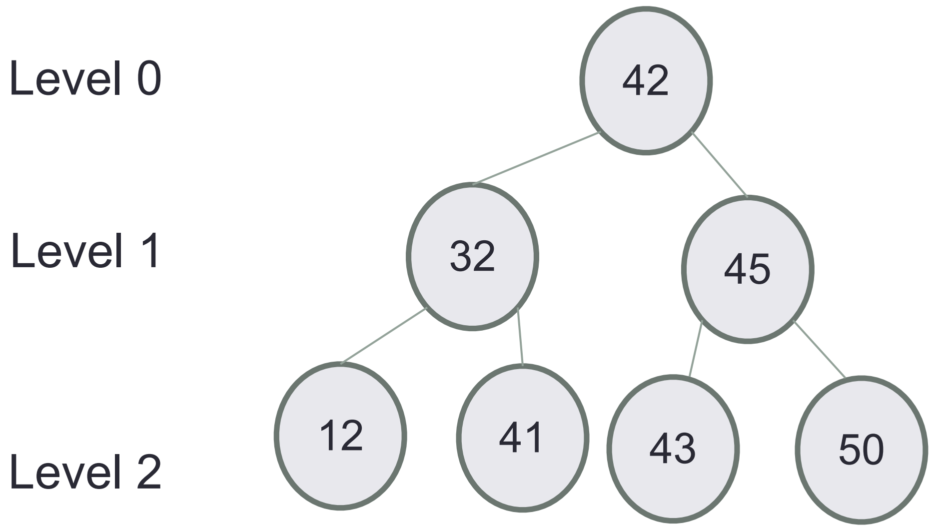
A. $O(1)$
B. $O(\log n)$
C. $O(n)$
D. $O(n^2)$
E. None of the above

- Path – a sequence of (zero or more) connected nodes.
- Length of a path - number of edges traversed on the path
- Height of node – Length of the longest path from the node to a leaf node.
- **Height of the tree** - Length of the longest path from the **root** to a leaf node.

BSTs of different heights are possible with the same set of keys

Examples for keys: 12, 32, 41, 42, 45

Types of BSTs

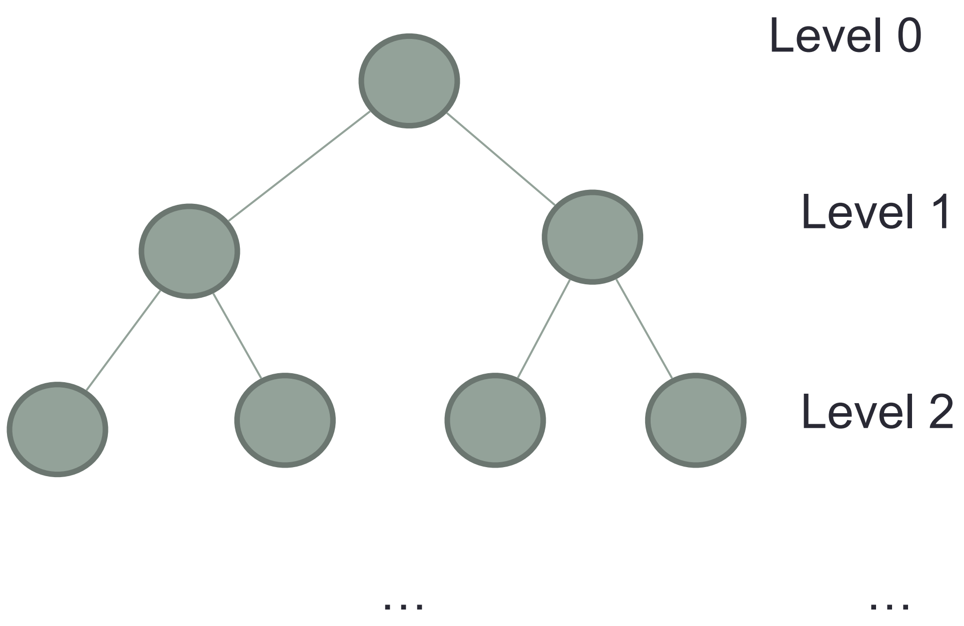


Balanced BST:

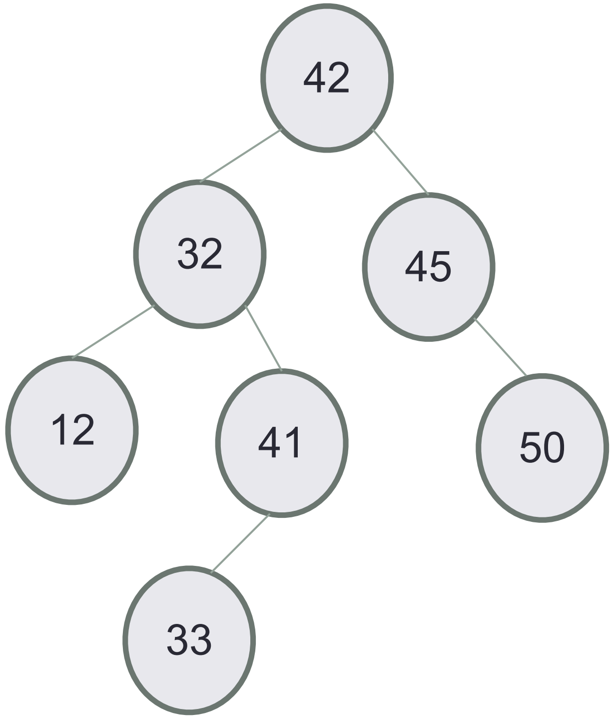
Complete Binary Tree: Every level, except possibly the last, is completely filled, and all nodes on the last level are as far left as possible

Full Binary Tree: A complete binary tree whose last level is completely filled

Relating H (height) and n (#nodes) for a full binary tree



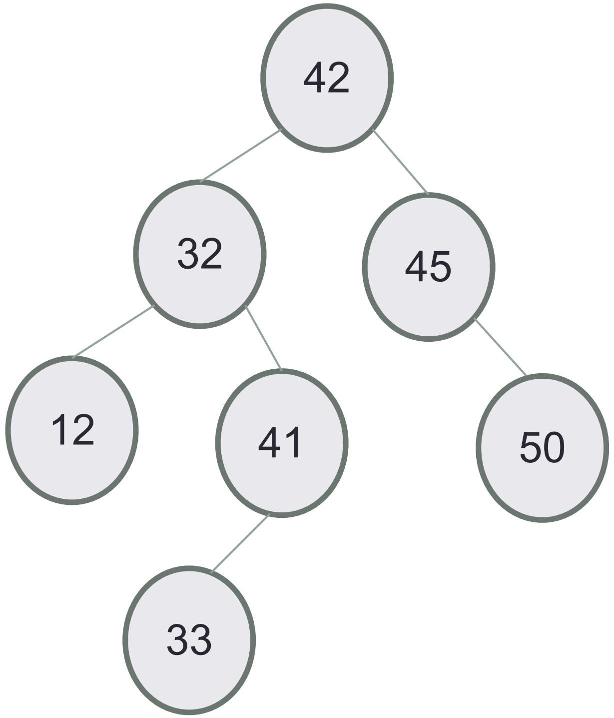
BST search - best case



Given a BST with N nodes, in the best case, which key would be searching for?

A. root node (e.g. 42)
B. any leaf node (e.g. 12 or 33 or 50)
C. leaf node that is on the longest path from the root (e.g. 33)
D. any key, there is no best or worst case

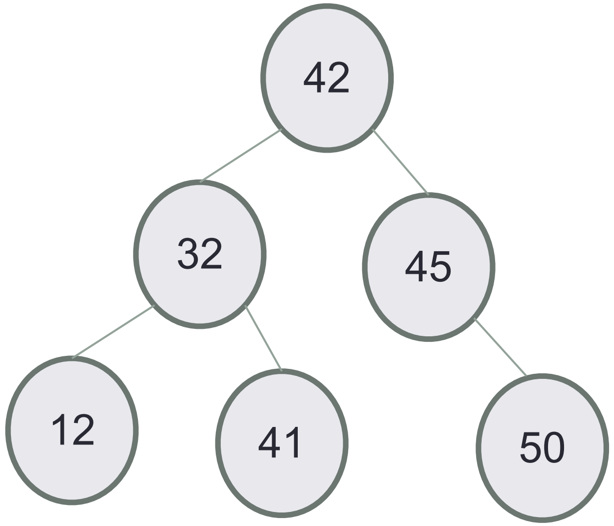
BST search - worst case



Given a BST with N nodes, in the worst case, which key would be searching for?

A. root node (e.g. 42)
B. leaf node (e.g. 12 or 41 or 50)
C. leaf node that is on the longest path from the root (e.g. 33)
D. a key that doesn't exist in the tree

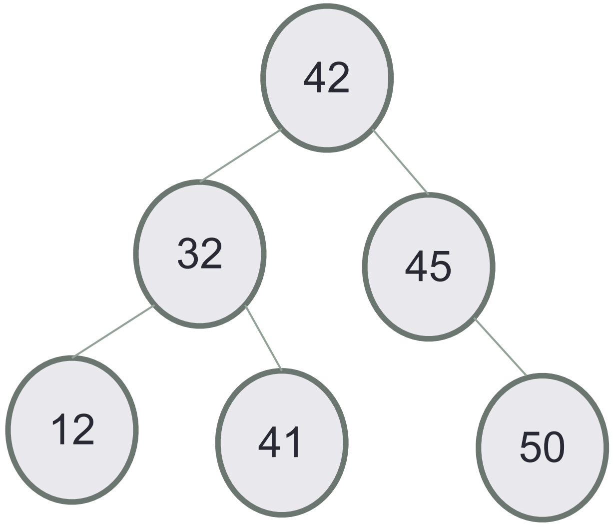
Worst case Big-O of search, insert, min, max



Given a BST of height H with N nodes, what is the running time complexity of searching for a key (in the worst case)?

A. O(1)
B. O(log H)
C. O(H)
D. O(H*log H)
E. O(N)

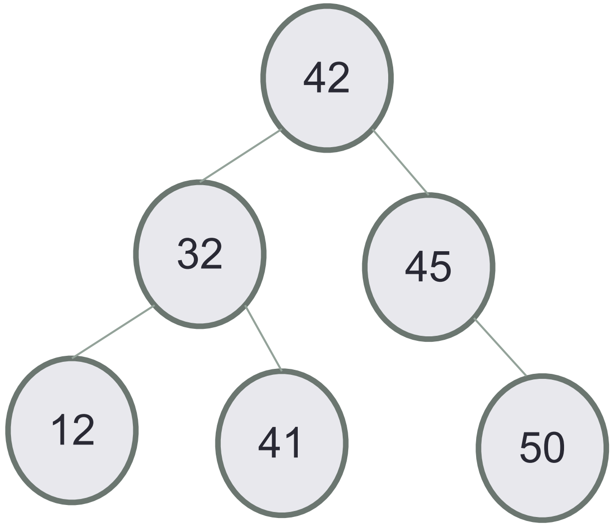
BST operations (worst case)



Given a BST of height H and N nodes, which of the following operations has a complexity of O(H)?

A. min or max
B. insert
C. predecessor or successor
D. delete
E. All of the above

Big O of traversals



In Order:

Pre Order:

Post Order:

```
procedure Convert(L: sorted linked list with n elements)
   Initialize an empty bst
   while (L is not empty)
       mid:= middle element of L
        remove mid from L
        insert mid to bst

return bst
```

Does the algorithm return a balanced BST for an input (sorted singly linked list) with n keys?

A. Yes
B. No

```
procedure Convert(L: sorted linked list with n elements)
   Initialize an empty bst
   while (L is not empty)
       mid:= middle element of L
       remove mid from L
       insert mid to bst
   return bst
```

What is the Big-O running time complexity of Convert?

```
void foo(int M, int N){
   int i = M;
   while (i >= 1) {
      i = i / 2;
   }
   for (int k = N ; k >= 0; k--){
    for (int j = 1; j < N; j = 2*j){
        cout << "Hello" << endl;
     }
   }
}
```

What is the Big-O running time of foo?

Balanced trees

- Balanced trees by definition have a height of O(log n)
- A completely filled tree is one example of a balanced tree
- Other Balanced BSTs include AVL trees, red black trees and so on
- Visualize operations on an AVL tree: https://visualgo.net/bn/bst