

## Lecture 7: Review of BST and Big O analysis

**Definition:** The **height** of a tree is the length of the longest path (number of edges) from the root to a leaf node.

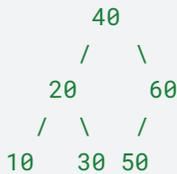
A single-node tree has height **0**. An empty tree has height **-1**.

**Definition:** A balanced BST is a BST whose height =  $O(\log n)$ , where  $n$  is the number of keys in the BST. Here we are thinking of height as a function of  $n$ , so  $h(n) = O(\log n)$

### Problem 1: Big-O Analysis of BST Operations

**Setup:** Put this BST on the board:

None



**Q: What is the height of this tree?**

Part A: Trace search

- Walk through searching for **key 30**. Write the path followed from root to 30.
- How is the running time of search related to the number of nodes visited?
- How is the number of nodes visited related to height of the BST(  $h$ )?

## Part B: Connecting height to Big-O

**Q: What is the worst-case running time of search in terms of  $h$  (height)?**

**Q: For a BST with  $n$  nodes, what are the best and worst possible heights?**

Draw best case and worst case with keys 10, 20, 30, 40, 50, 60, 70

**Takeaway:** BST operations (search, insert, min, max) are all  $O(h)$ . The height depends on insertion order. Worst case is  $O(n)$ , best case is  $O(\log n)$ .

## Problem 2: Writing a Recursive BST Function

Approach 1: Check immediate children (naive)

**First instinct:** At each node, just check that left child < node < right child.

None

```
isBST(node):
```

```
    if node is null, return true
```

```
    if left child exists and left->data >= node->data, return false
```

```
    if right child exists and right->data <= node->data, return false
```

```
    return isBST(left) AND isBST(right)
```

Does this work? Why or Why not?

Approach 2: Apply the full definition (bottom-up)

None

`isBST(node):`

`if node is null, return true`

`if max key in left subtree > node, return false`

`if min key in right subtree < node, return false`

`return isBST(left) AND isBST(right)`

Next Approach: Flip the direction — push constraints down (top-down)

**What extra parameters do I need?** A `min` and `max` defining the allowed range.

**How do they change?** Trace the ranges on a valid BST:

None



**Which traversal?**

Write the code

```
C/C++
class bst {
private:
    struct Node {
        int data;
        Node* left;
        Node* right;
    };
    Node* root;
    //Add helper here

public:
    bool isBST() const;
};
```

Link to LeetCode problem:

<https://leetcode.com/problems/validate-binary-search-tree/description/>

Helper needs (node, min, max). Check node against range, recurse with tighter bounds:

```
C/C++  
bool bst::isBST() const {  
    //TO DO  
  
}
```

Trace to verify

**Problem:** Return the k-th smallest key in the BST (1-indexed).

We need nodes in sorted order to know the k-th position → **inorder**. We pass a counter by reference that increments each time we visit a node. Once it hits k, we're done.

```
C/C++
void findKthSmallest(Node* r, int k, int& count, int& result) const {
    if (!r) return;

    findKthSmallest(r->left, k, count, result); // left first (smaller
keys)

    count++; // visit current node
    if (count == k) {
        result = r->data;
        return;
    }

    findKthSmallest(r->right, k, count, result); // right (larger keys)
}
```

**Takeaway:** Ask yourself — does the answer at this node depend on what's **above** it (constraints from ancestors → preorder) or what's **below** it (results from children → postorder)?